Influence of noise amplification on generation of regular short pulse trains in optical fibre pumped by intensity-modulated CW radiation

Sergey M. Kobtsev and Sergey V. Smirnov

Novosibirsk State University, Pirogova 2, Novosibirsk 630090, Russia
kobtsev@lab.nsu.ru

For the first time the influence of noise amplification on decay of modulated continuous-wave pumping into a pulse series in an optical fiber is considered. Dependence of noise-to-signal ratio in pulse train at fibre exit on initial modulation depth obtained both analytically and by means of numerical simulations. The minimum modulation frequency is estimated which leads to a regular pulse train formation from CW pumping.

OCIS codes: (190.0190) Nonlinear optics; (190.4370) Nonlinear optics, fibers

References and links

1. A. Hasegawa, "Generation of a train of soliton pulses by induced modulational instability in optical fibers," Opt. Lett. **9**, 288-290 (1984).
2. K. Tai, A. Tomita, J. L. Jewell, and A. Hasegawa, "Generation of subpicosecond solitonlike optical pulses at 0.3 THz repetition rate by induced modulational instability," Appl. Phys. Lett. **49**, 236-238 (1986).
3. E. M. Dianov, P. V. Mamyshev, A. M. Prokhorov, and S. V. Chernikov, "Generation of a train of fundamental solitons at a high repetition rate in optical fibers," Opt. Lett. **14**, 1008-1010 (1989).
4. S. V. Chernikov, E. M. Dianov, D. J. Richardson, R. I. Laming, and D. N. Payne, "114 Gbit/s soliton train generation through Raman self-scattering of a dual frequency beat signal in dispersion decreasing optical fiber," Appl. Phys. Lett. **63**, 293-295 (1993).
5. S. V. Chernikov, J. R. Taylor, and R. Kashyap, "Comblike dispersion-profiled fiber for soliton pulse train generation," Opt. Lett. **19**, 539-541 (1994).
6. M. Tadakuma, O. Aso, and S. Namiki, "A 104GHz 328fs soliton pulse train generation through a comb-like dispersion profiled fiber using short high nonlinearity dispersion shifted fibers," presented at OFC 2000.
7. S. Pitois, J. Fatome, and G. Millot, "Generation of a 160-GHz transform-limited pedestal-free pulse train through multiwave mixing compression of a dual-frequency beat signal," Opt. Lett. **27**, 1729-1731 (2002).
8. J. Fatome, S. Pitois, and G. Millot, "20-GHz-to-1-THz repetition rate pulse sources based on multiple four-wave mixing in optical fiber," IEEE J. Quantum Electron. **42**, 1038-1046 (2006).
9. J. M. Dudley and S. Coen, "Coherence properties of supercontinuum spectra generated in photonic crystal and tapered optical fibers," Opt. Lett. **27**, 1180-1182 (2002).
10. S. M. Kobtsev and S. V. Smirnov, "Coherent properties of super-continuum containing clearly defined solitons," Opt. Express **14**, 3968-3980 (2006).
11. F. Vanholsbeeck, S. Martin-Lopez, M. González-Herráez, and S. Coen, "The role of pump incoherence in continuous-wave supercontinuum generation," Opt. Express **13**, 6615-6625 (2005).
12. S. M. Kobtsev and S. V. Smirnov, "Modelling of high-power supercontinuum generation in highly nonlinear, dispersion shifted fibers at CW pump," Opt. Express **13**, 6912-6918 (2005).
13. N. Korneev, E. A. Kuzin, B. Ibarra-Escamilla, M. Bello-Jimènez, and A. Flores-Rosas, "Initial development of supercontinuum in fibers with anomalous dispersion pumped by nanosecond - long pulses," Opt. Express **16**, 2636-2645 (2008).
14. S. Martin-Lopez, A. Carrasco-Sanz, P. Corredera, L. Abrardi, M. L. Hernanz, and M. Gonzalez-Herraez, "Experimental investigation of the effect of pump incoherence on nonlinear pump spectral broadening and continuous-wave supercontinuum generation," Opt. Lett. **31**, 3477-3479 (2006).
15. J. H. Lee, Y. -G. Han, and S. Lee, "Experimental study on seed light source coherence dependence of continuous-wave supercontinuum performance," Opt. Express **14**, 3443-3452 (2006).
16. D. R. Solli, C. Ropers, P. Koonath and B. Jalali, "Optical rogue waves," Nature **450**, 1054-1058 (2007).
17. J. M. Dudley, G. Genty, and B. J. Eggleton, "Harnessing and control of optical rogue waves in supercontinuum generation," Opt. Express **16**, 3644-3651 (2008).
18. G. P. Agrawal, *Nonlinear Fiber Optics* (Academic Press, San Diego, California, 2001).

#93975 - $15.00 USD Received 17 Mar 2008; revised 28 Apr 2008; accepted 28 Apr 2008; published 7 May 2008

1. Introduction

The effect of generation of spectrally-broadened relatively short pulses in optical fibers under CW pump has certain specificity as compared to analogous effect under pulsed pump. One of the main features of spectrally-broadened radiation (SBR) under continuous pumping is a characteristic absence of any regularity in temporal distribution of its intensity in contrast to the SBR or super-continuum (SC) with periodic pulsed pump. Considerable spread in parameter values (energy, mean wavelength) and chaotic temporal distribution of SBR or SC under CW pump given rise to by the effect of modulation instability (MI) pose substantial limitations on applications of such radiation. For simplicity, we will name below both spectrally-broadened radiation and supercontinuum as SBR.

The goal of the present study is to explore possibilities of regularisation of the SBR temporal distribution under CW pumping by modulating the intensity of the pump radiation. Technically, modulation of CW radiation at the entrance to the optical fibre can be implemented either with the help of adequate high-frequency modulators or by producing beats between two or more monochromatic pump waves having relatively close frequencies. The latter approach was suggested by Hasegawa [1] and for the first time was implemented by Tai *et al* [2] as early as 1986. Subsequently, a series of papers were published on different modifications and development of this method (see, for instance, [3–7]). Specifically, in computer modelling [3] and experiment [4] it was demonstrated that application of optical fibres with adiabatically decreasing dispersion or active fibres allows generation of periodic trains of non-interacting solitons. Additionally, optical fibres with longitudinal step dispersion profile [5, 6] were used for generation of periodic soliton trains. In Ref [7] it was theoretically and experimentally demonstrated that generation of regular spectral-limited Gaussian pulses is possible in passive optical fibre with longitudinally constant dispersion provided the pumping wave has optimal power.

The initial, relatively small, intensity modulation of CW radiation may be amplified because of MI and the maximum of the frequency gain contour is a few hundred GHz away from the radiation frequency. As the modulation frequency is reduced down to dozens of GHz and less amplification of initial radiation intensity modulation drops considerably. However, it is this range of relatively low frequencies, which is most interesting for practical applications. In various papers dedicated to studies of decay of CW intensity-modulated radiation into pulse trains ultra-short pulses (USP) were generated with repetition frequency in the range between 60 GHz [5] and 340 GHz [2]. Achieved relatively high USP repetition rates constrain substantially the application field of USP/SBR generators with CW modulated pump. For example, metrological application requirement is that the pulse repetition rate should be measurable with radio-technical means. Lower pulse repetition rates may be necessary also for application of spectrally broadened radiation in time resolved spectroscopy. Thus, at pulse repetition rates around ~ 300 GHz the interval between the two neighbouring pulses (~ 3 ps) may be on the same order of magnitude or even less than the relaxation time of the investigated objects, therefore rendering such source useless for time resolved spectroscopy. In this connexion there is a problem of changing and lowering as far as possible the repetition frequency of generated pulses under CW modulated pump. The possibility of frequency detuning was already experimentally demonstrated earlier, *e.g.* in Ref [4] trains with repetition rate of 80 to 120 GHz were generated. Recently USP generation with much wider range of repetition rates from 20 GHz up to 1 THz was shown by Fatome, *et al* [8]. However, neither of publications we are aware of discusses the practically important issue of the limits to the range in which the detuning of USP repetition rate is possible under modulated CW pump. Influence of noise on the process of SBR generation has been studied in a series of papers. Thus for example it was shown that noise amplification results in coherence degradation of fs-pulse-pumped SBR [9,10] and plays a key role in SBR generation under CW pump [11,12] as well as under pump with nanosecond-long pulses [13]. The influence of CW pump

#93975 - $15.00 USD Received 17 Mar 2008; revised 28 Apr 2008; accepted 28 Apr 2008; published 7 May 2008

incoherence on SBR generation was studied in Refs [14, 15]. Most recently it was found that noise causes formation of extremely intensive fluctuations in SBR which are analogues of oceanic rogue waves [16,17]. However, as far as we know, there are no papers that discuss the influence of noise on generation of regular pulse train from modulated CW pump due to the effect of induced MI.

The present study focuses for the first time on the limiting conditions under which CW pump radiation with a certain intensity modulation decays into a regular pulse train, the repetition rate of which being determined by the modulation frequency. For the first time relative noise level in the periodical pulse train generated in an optical fibre is studied as dependent of the modulation depth and frequency. For the first time the characteristic range of minimum repetition rates is determined for pulses that can be formed in the decay process of modulated CW pumping.

2. Analytical study

Propagation of two monochromatic waves with close frequencies along an optical fibre will be accompanied (provided certain conditions are met) by growth of their beat amplitude and subsequent formation of a pulse train with the repetition frequency equal to the beat frequency between the two waves. (Analogous process must be observable when a single wave with initial amplitude modulation is fed into an optical fibre. In this case the spectrum will contain three rather than two equidistant lines, the rate of their intensity growth being identical to the case of two pumping lines.) The process of CW pump decay is mainly governed by four-wave mixing, what is corroborated by analytical estimates of parametric gain, as also by numerical modelling carried out taking into account different terms of the generalised non-linear Schrödinger equation (GNLSE). Resulting difference in GNLSE solutions obtained by inclusion of Kerr effect, stimulated Raman scattering, self-steepening as well as of second and third dispersion from solutions of NLSE (Kerr non-linearity + second dispersion) amounts to only about 1% for parameters corresponding to the experiment [2]. This allows considerable simplification of the analysis of the studied effect by using a simpler equation (NLSE), which has self-similar solutions. This allows us to study the propagation of a wave having fixed intensity level and fixed fibre parameters and then to scale the generated results to practically any interesting case by using variable substitutions $P \to kP$, $z \to z/k$, $t \to t/k^{1/2}$ $(\Omega \to \Omega\, k^{1/2})$. Therefore, unless otherwise stated, the results of numerical modelling correspond to the parameters of experiment [2]: combined power of the pumping waves at the beginning of the fibre $I_0 + I_s = 3$ W, dispersion and non-linear coefficients of the fibre $\beta_2 = -3.5$ ps^2/km and $\gamma = 1.8\times10^{-5}$ (cm·W)$^{-1}$ (non-linear fibre length $L_{NL} = 0.19$ km at $P_\Sigma = 3$ W).

Propagation of the pumping wave is controlled, aside from the input power, mainly by two parameters: modulation depth (equal to the ratio between the signal wave I_s and the pump wave I_0) and frequency detuning Ω of the signal wave from the pumping frequency. We will use a dimensionless coefficient ξ to quantify this detuning:

$$\xi = -\frac{\beta_2 \Omega^2}{2} \cdot L_{NL} \tag{1}$$

Where $L_{NL} = (\gamma P)^{-1}$ – non-linear fibre length. Note that $\xi = (\Omega/\Omega_c)^2$, where Ω_c is a frequency scale introduced in [18]. Analytical calculations [18] show that exponential growth in the beats amplitude will only happen if $0 < \xi < 2$, i.e. $\Omega^2 < 4/(-\beta_2 L_{NL})$. A stronger limit on the modulation frequency arises when a competing process is taken into account: the amplification of noise power (spontaneous radiation) because of MI. In order for the effect of induced MI to be observable at the exit from the fibre (at $z = z_*$) the amplitude of the signal wave has to be at least by a factor of η larger than the magnitude of amplified noise (η designates the minimal acceptable signal-to-noise ratio for the output radiation):

#93975 - $15.00 USD Received 17 Mar 2008; revised 28 Apr 2008; accepted 28 Apr 2008; published 7 May 2008

$$I_s e^{\frac{2z_*}{L_{NL}}\sqrt{2\xi-\xi^2}} > \eta \cdot I_{noise} e^{\frac{2z_*}{L_{NL}}} \tag{2}$$

By estimating the fibre length z_* from the condition that the intensity of the signal wave at the fibre exit (the first term in Eq. (2)) will be on the order of the pumping wave intensity I_0:

$$z_* = \frac{L_{NL}}{2\sqrt{2\xi-\xi^2}} \cdot \ln\frac{I_0}{I_s} \tag{3}$$

and substituting z_* for z in Eq. (2), we will obtain the condition on the parameter ξ:

$$|1-\xi| \;<\; \sqrt{1-\left(\frac{\ln\left(I_s/I_0\right)}{\ln\left(\eta I_{noise}/I_0\right)}\right)^2} \tag{4}$$

For typical parameters $I_0 \sim 3$ W, $I_s / I_0 \sim 10\%$, $\eta \sim 100$ we have $I_{\mathrm{noise}} / I_0 \sim 10^{-9}$ [12]: $|1-\xi| < 1-1.03\times10^{-2}$ from which at $\beta_2 = -3.5$ ps^2/km and $\gamma = 1.8\times10^{-5}$ $(\mathrm{cm}\cdot\mathrm{W})^{-1}$ ($L_{NL} = 0.19$ km) we find the range of initial frequency modulation that will be amplified because of instability and will lead to decay of CW radiation into a pulse train with the same repetition frequency: $\Omega/2\pi = 28 \dots 390$ GHz. For the same parameters and initial modulation depth of 0.1% we obtain $|1-\xi| < 0.90$ и $\Omega/2\pi = 86 \dots 381$ GHz. Note that the derived figures are approximations since in the conducted analytical consideration only initial phase of exponential growth of signal wave was taken into account. The error introduced in this approximation grows with initial modulation depth.

In order to study in more detail the influence of noise on decay of modulated CW pump we carried out a series of calculations based on numerical solution of non-linear Schrödinger equation. In Section 3 we will discuss the question of the upper integration limit in the propagation equation, and in Section 4 results generated in the course of modelling will be reported.

3. Modelling

We carried out numerical simulations for different values of parameter ξ and modulation depth I_s / I_0, located in the nodes of an orthogonal mesh. For each pair of parameters (ξ, I_s / I_0) NLSE was integrated with 100 different random instances of noise. Generated radiation spectra were than averaged and the signal-to-noise ratio calculated in the resulting spectrum as the ratio of spectral power in the centre of spectral lines and its value between the lines.

NLSE was integrated up to $z = z_{max}$, which was determined for each pair of parameter values (ξ, I_s / I_0) from the condition of maximum amplitude of pulses formed as after decay of the modulated CW pump. Optimal fibre length z_{max} depends both on the modulation depth of the CW pump and on dimensionless parameter ξ related to the modulation frequency. Dependence on the modulation depth obtained as a result of numerical modelling is very well approximated by a linear function of modulation depth logarithm (see Fig. 1), which is in full agreement with the approximation of exponential modulation growth we used to derive Eq. (2) and, further on, Eq. (4).

Identified with the help of numerical calculations dependence of the optimal fibre length z_{max} on parameter ξ at $I_s / I_0 = 10^{-3}$ is indicated in Fig. 2 with dotted line. The minimum in dependence $z_{max}(\xi)$ is achieved at $\xi = 1$ and corresponds to the maximum of the gain coefficient of MI, which leads to amplification of modulation amplitude and to formation of regular pulse train. For comparison, in the same graph an analytical estimate z_* given by Eq. (3) is shown by gray curve. Difference between z_* and z_{max} in Fig. 2 never exceeds 20%, indicating that the major contribution to the increase in the fibre length resulted from lowering ξ comes from decrease in the linear gain coefficient of MI [18].

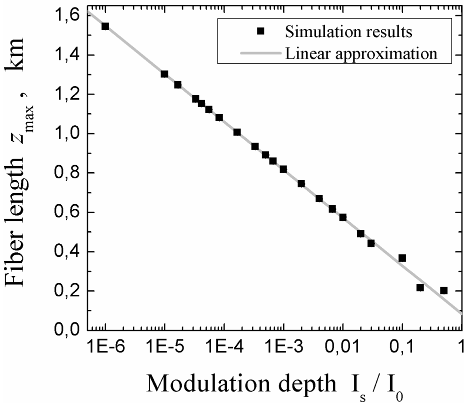

Fig. 1. Dependence of optimal fibre length z_{max} on modulation depth I_s/I_0 at ξ = 1.46 (corresponds to the experiments [2])

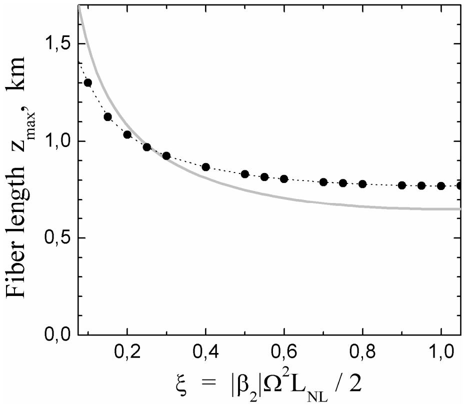

Fig. 2. Dependence of optimal fibre length z_{max} on ξ.

Figure 3 shows some examples of pulse trains and spectra obtained in numerical modeling at $\xi = 0.1$ and $I_s / I_0 = 1, 10^{-3}, 10^{-4}$. It can be seen that maximum depth of initial amplitude modulation gives rise to a regular pulse train that has a comb-like spectrum with low noise-to-signal ratio (NSR). When modulation depth is decreased to 10^{-3} the pulse train becomes slightly non-periodical what corresponds to a growth of noise background in the spectrum. Finally at $I_s / I_0 = 10^{-4}$ generated pulse train becomes fully stochastic and its spectrum lose it comb-like form. Similar pictures can be observed at different values of ξ, however each ξ is characterized by its own minimal initial modulation depth which still results in regular pulse train formation. Note that generated pulses have a pedestal with form and energy being functions of parameter ξ and I_s / I_0. As it has been shown earlier [7] this pedestal can be eliminated by appropriate choice of pumping parameters.

4. Results and discussion

In correspondence with analytical estimates given in Section 2 numerical modelling demonstrates that propagation of modulated CW pump along a fibre is accompanied by growth in modulation amplitude and, at certain values of parameters (ξ, I_s / I_0), the continuous wave is decomposed into a pulse train. The key radiation parameter here is the NSR: it defines periodicity of the generated train, amount of fluctuation in the pulse energy, ratio of “useful” pulse energy to the energy of the noise floor, degree of coherence in the generated radiation, and, ultimately, the very possibility to use a radiation source based on the chosen mode of spectral broadening in different applications. The dependence of the NSR upon the logarithm of the modulation depth and dimensionless parameter ξ is demonstrated in Fig. 4 as a contour plot: relative noise level is rendered in shades of gray and corresponding numeric values in dB are listed to the right of the plot beside the gray-scale chart.

#93975 - $15.00 USD Received 17 Mar 2008; revised 28 Apr 2008; accepted 28 Apr 2008; published 7 May 2008

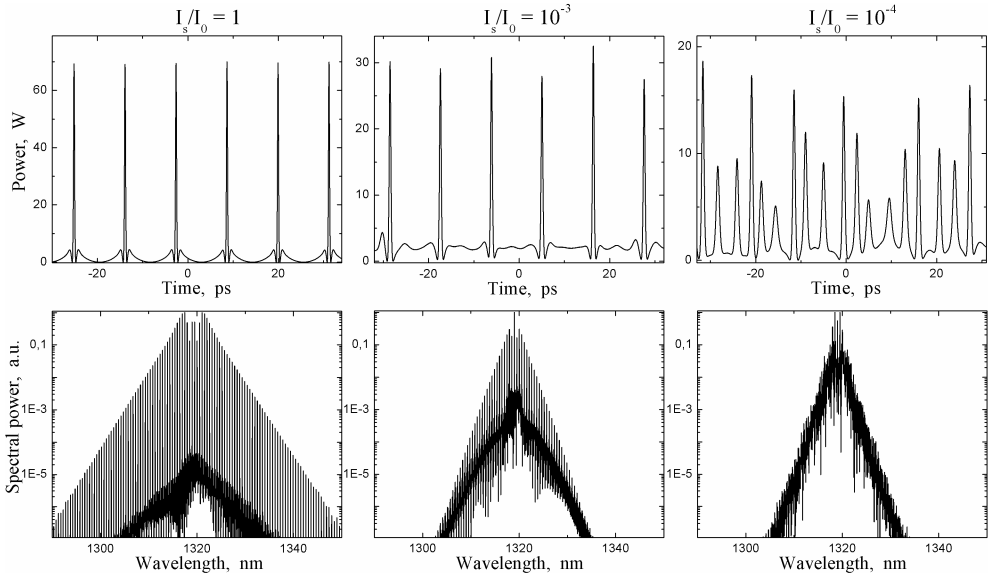

Fig. 3. Generated pulse train and its spectrum at $\xi = 0.1$ for different initial modulation depths I_s / I_0.

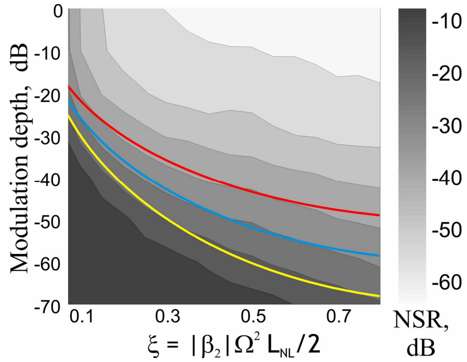

Fig. 4. Dependence of noise-to-signal ratio at fibre exit on parameter ξ and modulation depth.

It can be seen in Fig. 4 that the NSR reaches its minimum at large values of initial modulation depth (0.1–1) and large values of ξ (minimum is reached at $\xi = 1$, whereas the plot shows the range $0.08 < \xi < 0.8$). As the modulation depth is reduced and/or ξ is decreased the NSR worsens and reaches unity (0 dB) in the lower left part of the area plotted in Fig. 4, which corresponds to completely "noisy" generation mode and irregular temporal structure of the formed pulse train (as for $I_s / I_0 = 10^{-4}$ in Fig. 3).

For comparison, dependence $\xi(I_s / I_0)$ defined by Eq. (4) at $\eta = 10^2$, 10^3 and 10^4 is given in Fig. 4 in yellow, blue and red lines accordingly. It can be seen that these curves virtually coincide with the contour isolines of dependence NSR(ξ, I_s / I_0) obtained in numerical modelling, thereby confirming validity of approximations made while deriving Eq. (4).

Conducted numerical modelling provides means to estimate minimal repetition rate of pulses in a regular train that can be generated in decay process of modulated CW pump: $\xi_{\min} \sim 0.045$, which corresponds, in terms of experiment [2], to $\nu_{\min} = \Omega_{\min} / 2\pi \sim 59$ GHz. As it follows from Eq. (1), this value is inversely proportional to dispersion coefficient β_2 and hence, can be reduced by choosing a fibre with large absolute value of negative dispersion. For instance, the minimal repetition rate for the initial portion of fibre with dispersion $\beta_2 =$

–12.7 ps^2/km at 1.55 μm used in [4] amounts to $\nu_{\min}$ ~ 31 GHz at launched power P = 3 W, and for fibre used in [5] with β_2 = –12.7 ps^2/km at 1.55 μm this parameter is $\nu_{\min}$ ~ 27 GHz at the same power level.

Furthermore, as it follows from Eq. (1), frequency Ω of USP repetition can be also reduced by reduction of fibre non-linearity γ or by reducing radiation power P. The limiting factor in this case will be attenuation in the fibre. Thus, reduction of the radiation power will be accompanied by a proportional increase in the fibre length (3), along which the modulated pumping wave is disintegrated into a pulse train, this, in turn, leading to greater losses. For estimation of minimal level of launched power we will assume [18] that non-linearity in the fibre operates on effective length $z_{eff} \sim 1/\alpha$, where α is loss coefficient. Using effective fibre length z_{eff} instead of z_{max} and relation $z_{max} \sim L_{NL} / 0.3$ obtained as a result of numerical modelling carried out for $\xi_{\min} = 0.045$ and $I_s / I_0 = 1$ we find finally that $(L_{NL})_{\max} \sim 0.3 / \alpha$. For a fibre with attenuation of 0.2 dB/km $\alpha \sim 4.6 \times 10^{-2}$ km^{-1}, from which it follows that $(L_{NL})_{max} \sim$ 6.5 km. This means that lowering the launched power will allow a further reduction in USP repetition rate approximately by another factor of 30 as compared to figures specified above for L_{NL} = 0.19 km, that is down to the level of ~ 1 GHz and lower. Hence, choice of appropriate fibre and the launched power of CW pump allows generation of USP trains with repetition rates in the sub-GHz range. Nevertheless, it should be taken into consideration that lowering launched pump power also reduces the width of SBR spectrum. Significant reduction of pump power, therefore, will lead to relatively narrow spectrum of pulses exiting the fibre, which may also serve as a limiting factor for SBR/USP applications.

5. Conclusion

In summary, the present study for the first time reports results of analytical and numerical investigation into the impact of noise (spontaneous Raman scattering) upon the development of induced modulation instability. For the first time a condition on the frequency and depth of the initial modulation was defined, which together lead to disintegration of the CW pumping wave inside an optical fibre into a regular pulse train. Specifically, the minimal pulse repetition rate of the generated train and its dependence upon fibre parameters and radiation power was calculated. The results of numeric modelling carried out in the course of the study are in good agreement with analytical estimations we have derived, as well as with results of previous experimental research by other authors [1-7].

#93975 - $15.00 USD Received 17 Mar 2008; revised 28 Apr 2008; accepted 28 Apr 2008; published 7 May 2008