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Method for stabilization of the microwave modulation index in order to suppress the light shift of the coherent population trapping resonances

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Abstract. For the resonance of coherent population trapping (CPT), we show that in the case of a spatially inhomogeneous light shift (for example, due to the Gaussian transversal profile of the light beam intensity), the zero position of the error signal, formed by the use of phase-jump technique, depends on the integration time of the spectroscopic signal. Basing on this effect, we propose two-loop method to stabilize the microwave power at the point where the light shift vanishes.

1. Introduction

Microwave atomic clocks based on coherent population trapping (CPT) resonance [1, 2, 3, 4, 5, 6] are attractive for such applications as secure communications, global satellite navigation and positioning systems. The advantages of these devices is a fully optical excitation scheme of narrow rf resonance without a microwave cavity. This enables one to significantly reduce the dimension of the physical package (up to the chip-scale) and power consumption.

Spectroscopic measurements during the frequency stabilization are accompanied by a perturbation of clock transition frequency due to the ac-Stark effect. This light shift depends on the optical field intensity and is one of the key factors limiting the metrological characteristics (stability and accuracy) of atomic clocks. Various methods have been suggested to suppress light shift in CPT clocks based on Ramsey [7, 8, 9, 10, 11, 12, 13] or continuous-wave [14, 15, 16] spectroscopy. However, these methods require the use of an external laser power modulator, which limits the possibility of their implementation in chip-scale atomic clocks.

In present paper, we develop an alternative two-loop method for suppressing light shift in continuous-wave spectroscopy of CPT resonance in the case of spatially nonuniform profile of the light intensity. Our approach based on phase-jump spectroscopy [17] and power stabilization of the microwave signal (which is used to modulate the VCSEL current). The proposed method does not need a laser power modulator and is free from the problem of lineshape-asymmetry-induced shift (see more detail in Ref. [17]).



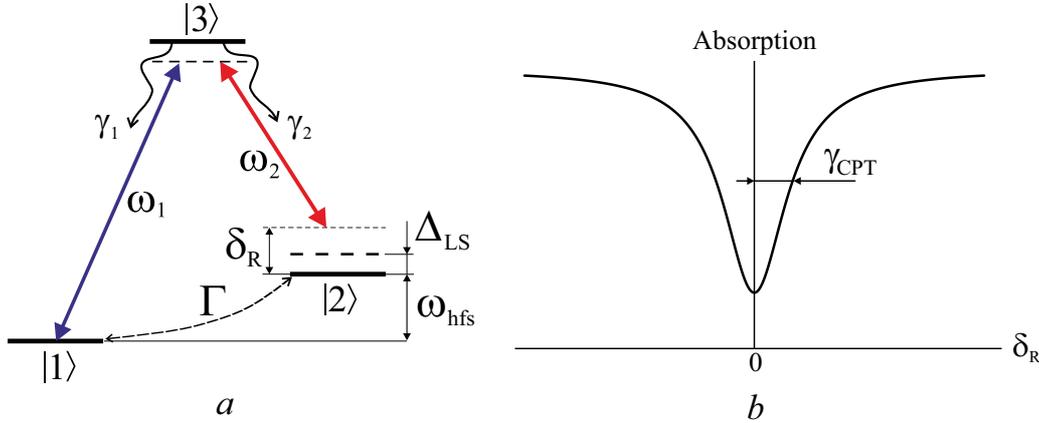


Figure 1. (a) The scheme of the three-level Λ system, where ω_1 and ω_2 are the frequencies of resonant optical fields; Δ_{sh} is light (Stark) shift of the clock transition frequency; γ_1 and γ_2 are the rates of spontaneous population transfer from upper state $|3\rangle$ to the lower states $|1\rangle$ and $|2\rangle$, respectively; Γ is decay rate of the coherence between the states $|1\rangle$ and $|2\rangle$. (b) CPT resonance in the absorption spectrum, where γ_{CPT} is half-width at half-maximum.

2. Theoretical model

As a theoretical model of the atomic medium, we consider a closed three-level Λ system (see figure 1a) interacting with a bichromatic light field:

$$E(t) = E_1 e^{-i[\omega_1 t + \varphi_1(t)]} + E_2 e^{-i[\omega_2 t + \varphi_2(t)]} + \text{c.c.}, \quad (1)$$

under conditions of coherent population trapping, that is, when the difference of frequencies $\omega_1 - \omega_2$ is scanned at the frequency ω_{hfs} of the transition between the lower states of the Λ system. We will describe the time dynamics of the Λ system using the formalism of an atomic density matrix, which has the following form in the basis of states $\{|j\rangle\}$:

$$\hat{\rho}(t) = \sum_{m,n} |m\rangle \rho_{mn}(t) \langle n|. \quad (2)$$

In the rotating wave approximation the density matrix elements satisfy the following equations:

$$\begin{aligned} \partial_t \rho_{31} &= (-\gamma_{opt} + i\delta_1) \rho_{31} + i\Omega_2 e^{-i\varphi_2(t)} \rho_{21} + i\Omega_1 e^{-i\varphi_1(t)} (\rho_{11} - \rho_{33}), \\ \partial_t \rho_{32} &= (-\gamma_{opt} + i\delta_2) \rho_{32} + i\Omega_1 e^{-i\varphi_1(t)} \rho_{12} + i\Omega_2 e^{-i\varphi_2(t)} (\rho_{22} - \rho_{33}), \\ \partial_t \rho_{21} &= [-\Gamma + i(\delta_R - \Delta_{LS})] \rho_{21} - i\Omega_1 e^{-i\varphi_1(t)} \rho_{23} + i\Omega_2 e^{i\varphi_2(t)} \rho_{31}, \\ \partial_t \rho_{11} &= -\Gamma (\rho_{11} - 1/2) + \gamma_1 \rho_{33} - i\Omega_1 e^{-i\varphi_1(t)} \rho_{13} + i\Omega_1 e^{i\varphi_1(t)} \rho_{31}, \\ \partial_t \rho_{22} &= -\Gamma (\rho_{22} - 1/2) + \gamma_2 \rho_{33} - i\Omega_2 e^{-i\varphi_2(t)} \rho_{23} + i\Omega_2 e^{i\varphi_2(t)} \rho_{32}, \\ \partial_t \rho_{33} &= -(\gamma_{sp} + \Gamma) \rho_{33} + i\Omega_1 e^{-i\varphi_1(t)} \rho_{13} - i\Omega_1 e^{i\varphi_1(t)} \rho_{31} + i\Omega_2 e^{-i\varphi_2(t)} \rho_{23} - i\Omega_2 e^{i\varphi_2(t)} \rho_{32}, \\ \rho_{12} &= \rho_{21}^*, \quad \rho_{13} = \rho_{31}^*, \quad \rho_{23} = \rho_{32}^*, \end{aligned} \quad (3)$$

under the condition of normalization (conservation of the total population):

$$\text{Tr}\{\rho\} = \rho_{11} + \rho_{22} + \rho_{33} = 1. \quad (4)$$

In equations (3) we use the following notations: $\Omega_1 = |d_{31} E_1|/\hbar$ and $\Omega_2 = |d_{32} E_2|/\hbar$ are Rabi frequencies for the transitions $|1\rangle \leftrightarrow |3\rangle$ and $|2\rangle \leftrightarrow |3\rangle$, respectively (d_{31} and d_{32} are matrix

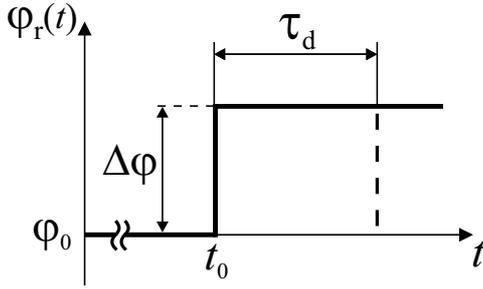


Figure 2. Scheme of jump modulation of the relative phase $\varphi_r = \phi_1 - \phi_2$ of the bichromatic field, φ_0 is initial phase difference of fields under stationary pumping of atoms, τ_d is detection time of the signal.

elements of the operator of electric dipole interaction); $\delta_1 = \omega_1 - \omega_{31}$ and $\delta_2 = \omega_2 - \omega_{32}$ are one-photon detunings of laser spectral components; $\delta_R = \delta_1 - \delta_2 = \omega_1 - \omega_2 - \omega_{\text{hfs}}$ is two-photon (Raman) detuning; Δ_{LS} is the light (Stark) shift of the clock transition frequency; γ_{opt} is the damping rate of optical coherence (due to spontaneous decay processes, collisions with buffer gas atoms, etc.); γ_1 and γ_2 are the rates of spontaneous population transfer from a state $|3\rangle$ to states $|1\rangle$ and $|2\rangle$, respectively; $\gamma_{\text{sp}} = \gamma_1 + \gamma_2$ is the spontaneous decay rate of the excited state $|3\rangle$; and the constant Γ describes the relaxation of atoms (for instance, due to transit effects) to an equilibrium isotropic distribution over the lower energy levels of the Λ system.

As a spectroscopic signal, we will study the absorbed power which, in the case of an optically thin medium, is proportional to

$$A(t) = \frac{\partial \rho_{33}}{\partial t} + (\gamma_{\text{sp}} + \Gamma)\rho_{33}. \quad (5)$$

Let us extract in the Rabi frequencies and light shift the spatial dependence of the transverse profile of the light beam f on the radius r :

$$\Omega_1^2(r) = \Omega_{10}^2 f(r), \quad \Omega_2^2(r) = \Omega_{20}^2 f(r), \quad \Delta_{\text{LS}}(r) = \Delta_0 f(r), \quad (6)$$

where Ω_{10} , Ω_{20} and Δ_0 are the Rabi frequencies and the light shift on the axis (i.e. at $r = 0$) of the light beam, respectively. In the case of a Gaussian light beam profile, the function $f(r)$ has the form

$$f(r) = e^{-r^2/r_0^2}, \quad (7)$$

where the radius r_0 determines the transverse size of the beam. To simulate the spectroscopic signal from the photodetector, it is necessary to integrate the local absorption (5) over the transverse profile of the laser beam:

$$\bar{A}(t) = \int_0^\infty \int_0^{2\pi} A(r, t) r d\varphi dr. \quad (8)$$

3. Phase-jumps spectroscopy

In this section, we briefly describe the main idea of the phase-jumps spectroscopy [17]. In this approach, the excitation of the dynamic (that is, time-dependent) response of the quantum system is carried out by modulating the relative phase of the bichromatic field

$$\varphi_r(t) = \varphi_1(t) - \varphi_2(t) \quad (9)$$

according to the jump law:

$$\varphi_r(t) = \begin{cases} \varphi_0, & \text{if } t < t_0; \\ \varphi_0 + \Delta\varphi, & \text{if } t \geq t_0, \end{cases} \quad (10)$$

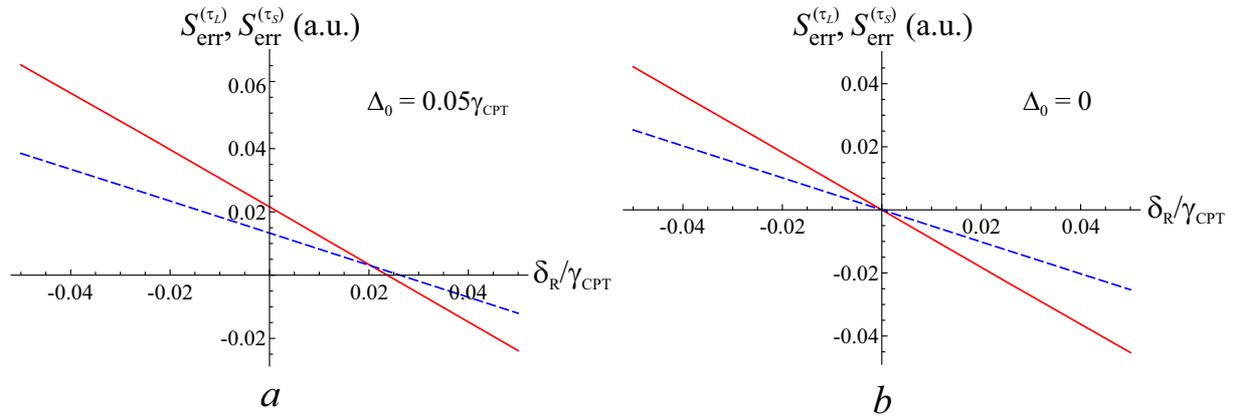


Figure 3. Error signals (11) for different detection times $\tau_L = 5\gamma_{\text{CPT}}^{-1}$ (solid red line) and $\tau_S = 1.5\gamma_{\text{CPT}}^{-1}$ (dashed blue line):

(a) CPT resonance is shifted, $\Delta_0 = 0.05\gamma_{\text{CPT}}$;

(b) light shift suppressed, $\Delta_0 = 0$.

Calculation parameters: $\Omega_{10}^2 + \Omega_{20}^2 = 0.01$, $\Omega_{10}/\Omega_{20} = 1.2$, $\gamma_{\text{opt}} = 50\gamma_{\text{sp}}$, $\gamma_1 = \gamma_2 = \gamma_{\text{sp}}/2$, $\Gamma = 10^{-4}\gamma_{\text{sp}}$, $\delta_1 = \delta_2 = 0.1\gamma_{\text{opt}}$.

where φ_0 is the initial phase difference of the two-frequency field, and $\Delta\varphi$ is the value of phase jump. A scheme of modulation of the relative phase φ_r is shown in figure 2. First the atoms are pumped to a steady state. Then the relative phase changes, in a jump, by some value, which causes a transition process in the absorption signal. At an exact two-photon resonance ($\delta_R = 0$) the dynamics of the transition process does not depend on the sign of the jump. However, at detuning from resonance ($\delta_R \neq 0$) the time evolution of the spectroscopic signal becomes different for phase jumps of opposite signs. The error signal for frequency stabilization can be generated using phase jumps of different signs ($\pm\Delta\varphi$) as follows:

$$S_{\text{err}}(\delta_R) = \int_{t_0}^{t_0+\tau_d} \bar{A}(t, +\Delta\varphi) dt - \int_{t_0}^{t_0+\tau_d} \bar{A}(t, -\Delta\varphi) dt, \quad (11)$$

where t_0 is the start time of the spectroscopic signal recording; τ_d is the duration of the signal accumulation (detection time). Thus, the error signal is defined as the difference between the area below the absorption curve for a positive jump of the relative phase and the area below the absorption curve for a negative phase jump. The frequency stabilization of the local oscillator is near the zero error signal: $S_{\text{err}}(\delta_R) = 0$. To maximize the error signal slope we need to use $\pm\pi/2$ phase jumps.

4. Method for stabilizing the microwave modulation index

In compact (including chip-scale) CPT atomic clocks, vertical-cavity surface-emitting laser (VCSEL) are used to generate a multi-frequency field. Harmonic modulation of the laser current using a signal from a microwave generator leads to phase modulation of the laser radiation, which corresponds to a set of equidistant coherent frequencies (harmonics) in the radiation spectrum. The modulation index is determined by the power of the microwave signal. The main contribution to the formation of the CPT state is introduced by two resonant field components. Nonresonant harmonics lead to a shift of the clock transition frequency due to the ac Stark effect. However, it is possible to select the modulation index of the laser radiation so that light shift is absent (see, for example, [18, 14, 19]).

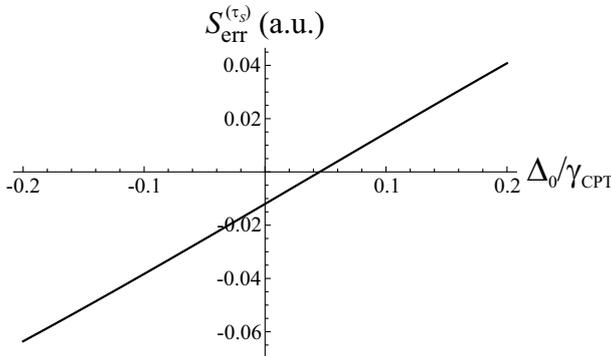


Figure 4. The error signal (11) with the short integration time τ_S as a function of the light shift at the center of the Gaussian profile Δ_0 . Calculation parameters: $\Omega_{10}^2 + \Omega_{20}^2 = 0.01$, $\Omega_{10}/\Omega_{20} = 1.2$, $\gamma_{\text{opt}} = 50\gamma_{\text{sp}}$, $\gamma_1 = \gamma_2 = \gamma_{\text{sp}}/2$, $\Gamma = 10^{-4}\gamma_{\text{sp}}$, $\delta_1 = \delta_2 = 0.1\gamma_{\text{opt}}$, $\tau_l = 5\gamma_{\text{CPT}}^{-1}$, $\tau_s = 1.5\gamma_{\text{CPT}}^{-1}$, δ_R is a solution to the equation $S_{\text{err}}^{(\tau_L)}(\delta_R) = 0$ at $\Delta_0 = 0.05\gamma_{\text{CPT}}$.

Using in the expression (11) two different integration times, long ($\tau_d = \tau_L$) and short ($\tau_d = \tau_S < \tau_L$), we can generate two error signals: $S_{\text{err}}^{(\tau_L)}(\delta_R)$ and $S_{\text{err}}^{(\tau_S)}(\delta_R)$, respectively. Figure 3 demonstrate the error signals calculated for $\tau_L = 5\gamma_{\text{CPT}}^{-1}$ and $\tau_S = 1.5\gamma_{\text{CPT}}^{-1}$, where γ_{CPT} is the half-width at half-maximum of the CPT resonance (see figure 1b). As can be seen from figure 3a, in the case of the light shift of the clock transition frequency, the zeros of the error signals are located at different two-photon detunings. Whereas in the absence of the light shift, the zeros of the error signals coincide, as shown in figure 3b. This feature can be used to organize a second feedback loop, which will stabilize the power of the microwave signal at a value where the light shift vanishes.

We propose the following algorithm for stabilizing the frequency of the local oscillator and the power of the microwave generator. For the error signal with the long integration time τ_L , the microwave power is fixed (that is, $P_{\text{mw}} = P_{\text{mw}}^{(fix)}$), and the variable detuning δ_R is stabilized at the zero of the first error signal:

$$S_{\text{err}}^{(\tau_L)}(\delta_R, P_{\text{mw}}^{(fix)}) = 0. \quad (12)$$

Then, measurements are carried out for the error signal with the short integration time τ_S , where the two-photon detuning found in the previous step is fixed (i.e. $\delta_R = \delta_R^{(fix)}$) and the microwave power P_{mw} is stabilized at the zero of the second error signal:

$$S_{\text{err}}^{(\tau_S)}(\delta_R^{(fix)}, P_{\text{mw}}) = 0. \quad (13)$$

Figure 4 shows the error signal with a short integration time $S_{\text{err}}^{(\tau_S)}$ as a function of the light shift Δ_0 at the center of the Gaussian profile determined by the microwave power P_{mw} . Repeating these iterations, we stabilize both parameters $\delta_R = \bar{\delta}_R$ and $P_{\text{mw}} = \bar{P}_{\text{mw}}$ which correspond to the solution of the following equations system:

$$S_{\text{err}}^{(\tau_L)}(\bar{\delta}_R, \bar{P}_{\text{mw}}) = 0, \quad S_{\text{err}}^{(\tau_S)}(\bar{\delta}_R, \bar{P}_{\text{mw}}) = 0. \quad (14)$$

In this case, our calculations show (see figure 3b) that $\bar{\delta}_R = 0$, i.e. the light shift of the stabilized clock frequency is absent.

5. Conclusion

It is shown that in the case of phase-jump spectroscopy of CPT resonance, the spatial inhomogeneity of the light shift (for example, due to the Gaussian transversal intensity profile of the light beam) leads to the dependence of the zero-point of the error signal on the integration

time. We propose to use this feature for active suppression of light shift by adjusting the microwave power. The criterion for the absence of a light shift is the condition when the zeros of the two error signals, generated at different integration times, coincide.

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References

- [1] Vanier J 2005 *Appl. Phys. B* **81** 421
- [2] Shah V and Kitching J 2010 *Adv. At. Mol. Opt. Phys.* **59** 21
- [3] Knappe S 2008 in *Comprehensive Microsystems* (Oxford: Elsevier) 571-612
- [4] Wang Z 2014 *Chin. Phys. B* **23** 030601
- [5] Kitching J 2018 *Appl. Phys. Rev.* **5** 031302
- [6] Carlé C, Petersen M, Passilly N, Abdel Hafiz M, de Clercq E and Boudot R 2021 *IEEE Trans Ultrason Ferroelectr Freq Control* **68** 3249
- [7] Yudin V I, Taichenachev A V, Basalaev M Yu, Zanon-Willette T, Pollock J W, Shuker M, Donley E A and Kitching J 2018 *Phys. Rev. Applied* **9** 054034
- [8] Yudin V I, Taichenachev A V, Basalaev M Yu, Mehlstäubler T E, Boudot R, Zanon-Willette T, Pollock J W, Shuker M, Donley E A and Kitching J 2018 *New J. Phys.* **20** 123016
- [9] Abdel Hafiz M, Coget G, Petersen M, Rocher C, Guérandel S, Zanon-Willette T, de Clercq E and Boudot R 2018 *Phys. Rev. Applied* **9** 064002
- [10] Abdel Hafiz M, Coget G, Petersen M, Calosso C E, Guérandel S, de Clercq E and Boudot R 2018 *Appl. Phys. Lett.* **112** 244102
- [11] Shuker M, Pollock J W, Boudot R, Yudin V I, Taichenachev A V, Kitching J and Donley E A 2019 *Phys. Rev. Lett.* **122** 113601
- [12] Shuker M, Pollock J W, Boudot R, Yudin V I, Taichenachev A V, Kitching J and Donley E A 2019 *Appl. Phys. Lett.* **114** 141106
- [13] Basalaev M Yu, Yudin V I, Kovalenko D V, Zanon-Willette T and Taichenachev A V 2020 *Phys. Rev. A* **102** 013511
- [14] Shah V, Gerginov V, Schwindt P D D, Knappe S, Hollberg L and Kitching J 2006 *Appl. Phys. Lett.* **89** 151124
- [15] Yudin V I, Basalaev M Yu, Taichenachev A V, Pollock J W, Newman Z L, Shuker M, Hansen A, Hummon M T, Boudot R, Donley E A and Kitching J 2020 *Phys. Rev. Applied* **14** 024001
- [16] Abdel Hafiz M, Vicarini R, Passilly N, Calosso C E, Maurice V, Pollock J W, Taichenachev A V, Yudin V I, Kitching J and Boudot R 2020 *Phys. Rev. Applied* **14** 034015
- [17] Basalaev M Yu, Yudin V I, Taichenachev A V, Vaskovskaya M I, Chuchelov D S, Zibrov S A, Vassiliev V V and Velichansky V L 2020 *Phys. Rev. Applied* **13** 034060
- [18] Zhu M and Cutler L S 2000 *Proceedings of 32th Annual Precise Time and Time Interval Systems and Applications Meeting* (Reston: Institute of Navigation) p 311
- [19] Yudin V I, Taichenachev A V and Basalaev M Yu 2016 *Phys. Rev. A* **93** 013820