

# Efficiency of different methods of extra-cavity second harmonic generation of continuous wave single-frequency radiation

SERGEY KHRIPUNOV,<sup>1,\*</sup> SERGEY KOBTSEV,<sup>1,2</sup> AND DABA RADNATAROV<sup>1</sup>

<sup>1</sup>Division of Laser Physics and Innovative Technologies, Novosibirsk State University, Pirogova Str., 2, Novosibirsk 630090, Russia

<sup>2</sup>Tekhnoscan-Lab LLC, Inzhenernaia Str., 26, Novosibirsk 630090, Russia

\*Corresponding author: khripunovsa@gmail.com

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**This work presents for the first time to the best of our knowledge a comparative efficiency analysis among various techniques of extra-cavity second harmonic generation (SHG) of continuous-wave single-frequency radiation in nonperiodically poled nonlinear crystals within a broad range of power levels. Efficiency of nonlinear radiation transformation at powers from 1 W to 10 kW was studied in three different configurations: with an external power-enhancement cavity and without the cavity in the case of single and double radiation pass through a nonlinear crystal. It is demonstrated that at power levels exceeding 1 kW, the efficiencies of methods with and without external power-enhancement cavities become comparable, whereas at even higher powers, SHG by a single or double pass through a nonlinear crystal becomes preferable because of the relatively high efficiency of nonlinear transformation and fairly simple implementation.** © 2016 Optical Society of America

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## 1. INTRODUCTION

Second harmonic generation (SHG) is often required when fundamental output with analogous parameters is not available for the target spectrum. There are many methods of SHG featuring different efficiencies, implementation complexities, and requirements for the input radiation and nonlinear medium parameters [1–3]. Since generation of the SH is a nonlinear process, it is generally more efficient when the power density within the nonlinear medium and the medium nonlinearity are higher. Intuitively, in a medium with a given nonlinearity, pulsed radiation with relatively high peak power should be more efficiently converted than continuous-wave (CW) radiation. However, a comparatively broad spectrum of pulsed radiation requires a correspondingly broad phase matching bandwidth of the nonlinear medium, which is achieved, as a rule, by shortening the nonlinear crystal and therefore reducing SHG efficiency. As a consequence, today's best implementations, for instance, of extra-cavity SHG, demonstrate an efficiency of 70% for pulsed radiation [4], whereas SH conversion of 98% has been demonstrated for CW radiation [5]. It should be noted that the best results in both cases were achieved in external enhancement cavities [6,7] containing a nonlinear crystal and providing a considerable amplification

of laser radiation intensity within the high-Q external optical cavity. The power-enhancement factor in an external cavity may be as high as 150–200 for CW radiation [7,8]. As an illustration, a CW input power of 1–2 W leads to an intra-cavity power level of 100–300 W. Obviously, given relatively low intensities of input CW radiation, external enhancement cavities allow for a considerable improvement of SHG efficiency compared to SHG in a single pass (SP) and a double pass (DP) through a nonlinear crystal [9–11]. However, as the fundamental radiation intensity grows, so does SP and DP SHG efficiencies, whereas the efficiency of SH conversion in an external cavity stays close to the attainable maximum and has practically no potential for improvement. Correspondingly, the difference in efficiency between external-cavity SHG and the SH generated in a SP or DP of radiation through a nonlinear crystal will grow smaller as the fundamental radiation intensity is increased.

The present work analyzes for the first-time to the best of our knowledge the efficiency of external-cavity SHG, as well as that of SP and DP SHG in a broad range of input powers of fundamental CW laser radiation. In a previous paper, a comparative analysis of SHG efficiency was carried out for an external power-enhancement cavity and intra-cavity frequency doubling [12]. This paper discussed SHG efficiency in relation

to various methods of extra-cavity frequency doubling, which do not necessitate modification of the laser optical layout within the range of fundamental CW radiation powers up to 10 kW (this figure corresponds approximately to the output power limit of modern commercial single-mode fiber lasers [13]). In the following sections, we will consider three typical implementations of extra-cavity frequency doubling: (1) SP CW SHG in a conventional [nonperiodically poled (PP)] nonlinear crystal; (2) DP CW SHG in a conventional (non-PP) nonlinear crystal; and (3) CW SHG in an external cavity (“multi-pass” SHG) with a conventional (non-PP) nonlinear crystal.

Here, we will not discuss any configurations relying on PP nonlinear crystals since they have a much lower optical damage threshold (photorefractive damage threshold not exceeding a few MW/cm<sup>2</sup> [14]) compared to conventional nonlinear crystals (typical damage threshold ranging between 1 and 100 GW/cm<sup>2</sup>) [2,15] and therefore cannot be used at relatively high incident radiation powers. Another problem typically suffered by PP nonlinear crystals is unstable SHG even at power levels not leading to damage. Other techniques left out of this study are those based on waveguide crystals (because the SH power saturates at comparatively low fundamental wave powers) and cascaded multi-crystal layouts (because of issues with the SH beam quality).

Our work on a comparative analysis of SHG efficiency in external cavities and in SP or DP configurations within a broad range of input fundamental radiation powers was motivated by recent publications [16,17]. These articles reported SP SHG efficiency of several tens of percent with both pulsed and CW fundamental radiation at average powers of 930 W [16] (SHG efficiency 46.8%), 1067 W [17] (SHG efficiency 51.5%), and 2300 W [18] (SHG efficiency 34.8%) with pulsed radiation and 1035 W [17] (SHG efficiency 34.4%) with CW radiation.

The key goal of our work is to find out how the efficiency of CW single-frequency SHG is compared in various configurations over as wide a power range as possible. To ensure the validity of the comparison among specific configurations (or methods) of extra-cavity SHG, a single type of nonlinear crystal (LBO) with a single fixed fundamental wavelength (1064 nm) was used in all analyzed configurations.

We should point out that the following analysis in relation to power-enhancement cavities will be limited to external cavities only. SHG configurations with power-enhancement cavities inside the laser resonator [19] or in a coupled resonator [20] are still rare and will not be included in the scope of the work.

## 2. THEORY

Schematic diagrams of the studied techniques of extra-cavity SHG of CW single-frequency radiation are shown in Fig. 1: frequency doubling in a SP of the fundamental radiation through a nonlinear crystal [Fig. 1(a)], analogous DP configuration [Fig. 1(b)], and frequency doubling in an external resonant cavity [Fig. 1(c)]. The analytical formula for SHG efficiency in all included methods was derived from the basis of the Boyd–Kleinman theory for SHG with circular Gaussian

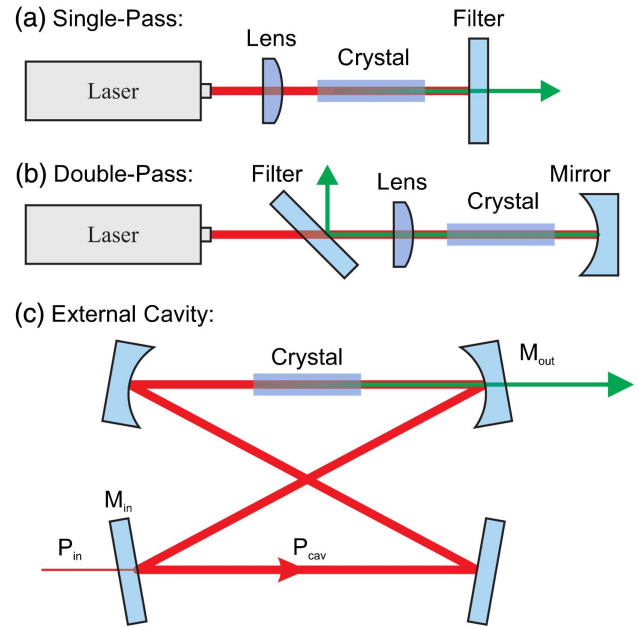


Fig. 1. Schematic diagrams of optical frequency doubling methods.

beams [21,22] complemented by a theory taking into account pump depletion in the case of high SHG conversion efficiency [23]. Double-pass treatment also allowed for the interference between the forward- and backward-propagating waves. In calculations for the external resonant cavity, we applied the Boyd–Kleinman theory and took into consideration the pump power depletion.

### A. Single-Pass Frequency Doubling in a Nonlinear Crystal

In this case, the following expressions can be used for SH power  $P_{2\omega,SP}$  and SHG efficiency  $\eta_{SP}$  as functions of the fundamental wave power [23]:

$$P_{2\omega,SP} = P_{1\omega}\eta_{SP}, \quad \eta_{SP} = \tanh^2 \sqrt{\gamma_{SHG} P_{1\omega}}, \quad (1)$$

where

$$\gamma_{SHG} = \frac{2\omega_1^3 d_{eff}^2}{\pi \epsilon_0 c^4 n^2} \times l \times h(\sigma, B, \xi),$$

$c$  is the speed of light in vacuum,  $\omega_1$  is the fundamental radiation frequency,  $d_{eff}$  is the effective coefficient of nonlinear quadratic polarization,  $\epsilon_0$  is the electric constant,  $n = n(\omega) \cong n(2\omega)$  is the refractive index of the medium for the fundamental harmonic,  $l$  is the optical crystal length, and  $h(\sigma, B, \xi)$  is the Boyd–Kleinman function depending on the parameters of radiation focusing in the crystal and in general given by the integral

$$h(\sigma, B, \xi) = \frac{1}{4\xi} \int_{-\xi}^{\xi} \int_{-\xi}^{\xi} \frac{e^{i\sigma(\tau-\tau')} e^{-B^2(\tau-\tau')/\xi}}{(1+i\tau)(1-i\tau')} d\tau d\tau',$$

where  $\xi = l/b$  is the focusing parameter;  $\sigma = b\Delta k/2$  is the phase mismatch parameter;  $b = 2\pi n\omega_0^2/\lambda$  is the confocal parameter of a beam with the waist radius  $w_0$ . In  $\Delta k = |2k_1 - k_2|$ ,  $k_1$  and  $k_2$  are the wave vectors for the fundamental

and the harmonic, respectively; and in  $B = \rho\sqrt{|k_1|}/2$ ,  $\rho$  is the walk-off angle.

### B. Double-Pass Frequency Doubling in a Nonlinear Crystal

Radiation frequency doubling in a nonlinear crystal in two passes can be calculated using Expression (1) and taking into account the phase of the SH radiation entering the crystal after reflection from the mirror [10]

$$A_{DP} = A_1 e^{i\phi} + A_2,$$

where  $A_1$  and  $A_2$  are the SH wave amplitudes after the first and the second pass through the nonlinear crystal, respectively. According to [10], the maximum DP efficiency is reached in the absence of linear losses at  $\phi = \pi$

$$\eta_{DP} = \tanh^2 \sqrt{4\gamma_{SHG} P_{1\omega}}.$$

### C. Frequency Doubling in an External Cavity with a Nonlinear Crystal

To calculate SHG efficiency for an external resonant cavity, Expression (1), where the fundamental radiation power  $P_{1\omega}$  is taken as the intra-cavity value, can be used again. Radiation losses  $L$  in the cavity are composed of the losses at the cavity mirrors and the SH conversion losses.

Let input mirror  $M_{in}$  [Fig. 1(c)] have transmittance  $T$  and total intra-cavity losses  $L$ . Then, according to [24], the highest intra-cavity power is reached when  $T = L$ , and the ratio of the intra-cavity power  $P_{cav}$  to the incident radiation power  $P_{in}$  can be written as

$$\frac{P_{cav}}{P_{in}} = \frac{\eta_{SP}(P_{cav}) \times P_{cav}}{P_{in}}.$$

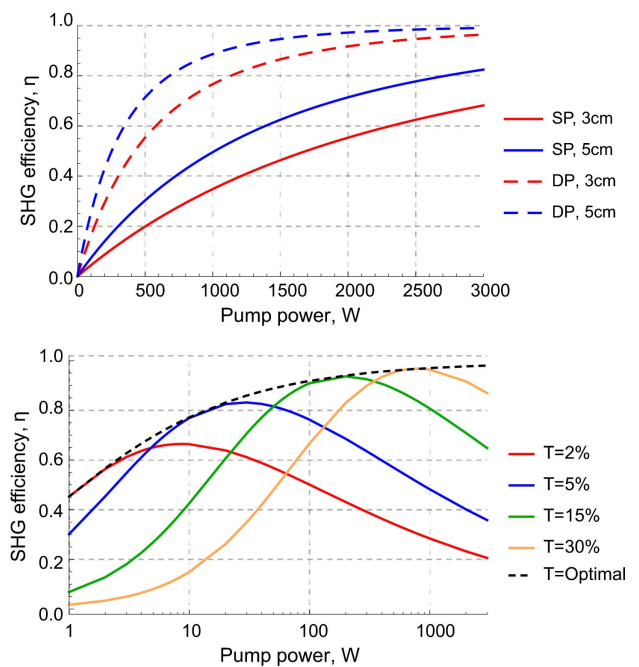
In calculating SHG efficiency for all the mentioned configurations, we used the value of effective crystal nonlinearity  $d_{eff} = 1$  pm/V; this corresponds to the LBO crystal and frequency doubling of a 1064 nm wave under noncritical phase matching conditions. We also assumed that the fundamental beam inside the crystal is focused so that the value of its confocal parameter equals the length of the crystal (the corresponding value of the Boyd–Kleinman function  $b(\sigma, B, \xi) = 0.8$ ).

## 3. RESULTS OF SHG EFFICIENCY CALCULATIONS IN THE STUDIED CONFIGURATIONS

Our calculations carried out on the basis of the formula and conditions detailed in the previous section allowed the comparison of SHG efficiency in the three studied configurations.

Figure 2 (top) demonstrates the dependencies of SHG efficiency in SP and DP configurations computed for the lengths of the nonlinear LBO crystal of 3 and 5 cm within the fundamental wave power range of 1 W to 3 kW. It can be seen that both SP and DP configurations attain efficiency in the tens of percent at the input power of several hundred watts. At the input power of about 1 kW, the SHG efficiency in the SP configuration may reach 50% and 90% in DP frequency doubling.

Dependencies of SHG efficiency in an external resonant cavity upon the input radiation power are presented in

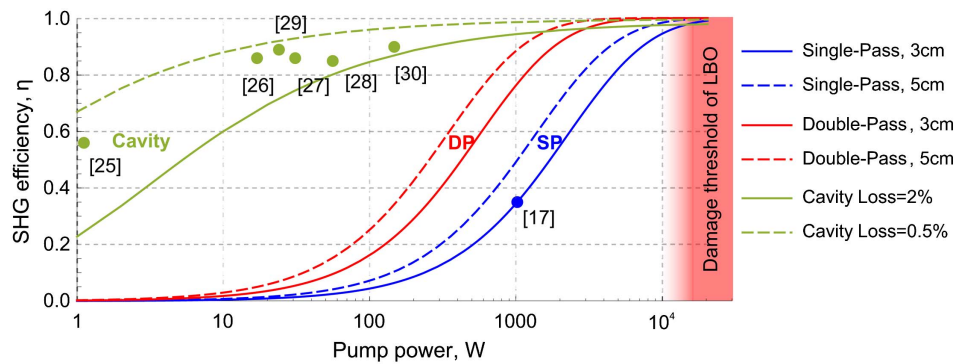


**Fig. 2.** (Top) SHG efficiency in SP and DP configurations with 3-cm-long and 5-cm-long LBO crystals; (bottom) SHG efficiency in an external resonant cavity as a function of the input radiation power at different values of the input mirror transmittance  $T$ .

Fig. 2 (bottom) under the assumption that the fundamental radiation losses at the mirrors are equal to 1%. Colored curves in Fig. 2 (bottom) correspond to SHG efficiency at different values of transmittance  $T$  of the input mirror; and the black dashed curve corresponds to SHG efficiency at the optimal transmittance of the input mirror equal to the total optical losses in the cavity, including conversion into SHG, which depends nonlinearly on the input radiation power.

SHG efficiency dependencies in all three studied configurations are summarized in Fig. 3. For the configuration with an external resonant cavity, the input mirror transmittance was taken at the optimum for each value of the input radiation power, whereas the optical losses in the cavity were assumed to be 0.5% and 2%. It can be seen from Fig. 3 that the DP configuration reaches a similar SHG efficiency as the external-cavity one at the fundamental radiation powers of 2–3 kW. The SP configuration “catches up” with the external cavity at higher power levels of  $\sim 10$  kW.

As the input radiation power approaches 10 kW, the efficiency of all studied SHG configurations converges to 100%, but this power level is close to the optical damage threshold of the nonlinear crystal (under optimal beam focusing conditions). It must be noted that both SP and DP configurations may become feasible even at relatively low fundamental radiation power of  $>100$  W. Both SP and DP configurations are considerably less efficient than the external-cavity one in the input power range of 100 W to 1 kW. Nevertheless, relative simplicity of these two approaches and their achievable efficiency of tens of percent may give them a practical advantage over a considerably more complicated and expensive external-cavity configuration. Furthermore, the SP and DP



**Fig. 3.** SHG efficiency in different frequency doubling configurations as a function of the input radiation power. Data taken from respective publications are marked with solid circles.

configurations have an additional benefit of not requiring single-frequency input: the fundamental radiation may contain multiple frequencies as long as its total spectrum does not exceed the spectral width of the nonlinear crystal's phase matching.

We should also note that although our study was conducted with a specific nonlinear crystal (LBO) at a particular CW radiation wavelength (1064 nm), the generated results are of a more general value. The calculated data range of Fig. 3 covers experimental measurements taken at other radiation wavelengths (Ti:sapphire laser [25], Raman fiber systems [26–28], Yb fiber laser [29], and Nd laser [30]) with other nonlinear crystals and even with long-pulsed lasers [17]. This is an indication that our modeling was carried out on a realistic range of data determined, for example, in the case of an external cavity by the cavity optical losses within 0.5%–2%.

#### 4. CONCLUSION

The present work for the first time to the best of our knowledge offers a comparative study of three different approaches to SHG of CW single-frequency radiation, which identifies input radiation power ranges where efficiency of SP and DP configurations increases to a practically feasible level of tens of percent and then attains performance comparable to that of SHG configurations with an external power-enhancement cavity. Both SP and DP approaches may become preferable at input powers exceeding 100 W, even though their efficiency is considerably lower than that of external-cavity systems because of their simplicity and tolerance to non-single-frequency input. At the fundamental radiation power in the range of 1–10 kW, SP and DP configurations compete in efficiency with external-cavity SHG.

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