Modelling of noise-like pulses generated in fibre lasers

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ABSTRACT

The present paper for the first time proposes and studies a relatively simple model of noise-like pulses that matches the experimental data well and suggests that there is a correlation between phases of adjacent spectral components of noise-like pulses. Comparison of a relatively basic model of 'random' pulses with the results of noise-like pulse modelling in mode-locked fibre lasers based on coupled non-linear Schrödinger equations demonstrates that it adequately reproduces temporal and spectral properties of noise-like pulses as well as correlation between adjacent modes so that it's possible to use the proposed model for highly efficient simulations of promising applications of noise-like pulses, such as material processing, non-linear frequency conversion, microscopy, and others.

Keywords: noise-like pulses, double-scale pulses, mode-locked laser, fibre laser, ultrafast laser, non-linear polarisation evolution, femtosecond clusters

1. INTRODUCTION

Recently it has been shown that fibre lasers mode-locked by means of non-linear polarisation evolution (NPE) effect support large variety of lasing regimes which may differ from each other both qualitatively[1, 2] and quantitatively[3]. Among the fascinating diversity of lasing regimes found in NPE-lasers[1, 4], much attention is paid to so-called noise-like pulses[5, 6] which are also known as double-scale optical lumps[2] or femtosecond clusters[7]. These pulses may be considered as wave-packets containing a large number of sub-pulses with different amplitude and phase[7]. Parameters of these sub-pulses (such as peak power, duration, energy, or instantaneous frequency) may experience significant fluctuation both during a single noise-like pulse and from one such pulse to another. Coherence time of noise-like pulses is determined by the sub-pulse duration, which may be substantially shorter than that of the wave packet as a whole. As a result, such pulses have two different typical duration scales clearly visible on the intensity auto-correlation function (ACF) whose specific shape is a characteristic 'fingerprint' of noise-like pulses. Namely, ACF of such pulses has a very distinct narrow central peak sitting on top of a broad pedestal. The width of this narrow peak corresponds to the typical sub-pulse duration and usually amounts to several hundred femtoseconds.

Noise-like or double-scale pulses were mentioned for the first time at the close of the 1990's[5], when pulses with broad spectrum and noise-like behaviour of their intensity and phase were generated in a mode-locked erbium laser. For a long time, noise-like pulses were disregarded as 'not sufficiently coherent' and thus did not draw appreciable attention to the respective laser generation regimes. Nevertheless, over recent years, this topic has been rapidly gaining popularity[8] after the observation of noise-like pulses with relatively high energy in ultra-long fibre lasers. Active research prompted by this discovery has shown that noise-like pulses may even be preferable to 'regular' ones in a number of applications due to their particular properties such as high peak power and low coherence time. The list of promising applications of noise-like pulses includes non-linear frequency conversion, such as harmonic generation[9], Raman conversion[10, 11] and super-continuum (SC) generation[12-17], as well as applications in imaging and sensing systems with high temporal and/or spatial resolution. Elevated noise levels intrinsic to noise-like pulses may pose an obstacle to pulse compression[6, 18]. However, the noise is known to play an important role in super-continuum generation[19-22] and Raman conversion including soliton self-frequency shift effect[23-26] where noise may not only be a negative factor deteriorating SC coherence but may also play a positive role strengthening spectral broadening. It was furthermore demonstrated that generation of noise-like pulses in long lasers represents a remarkably multiform phenomenon involving many non-linear optical mechanisms whose interplay may result in the emergence of diverse spatio-temporal coherent structures in laser radiation[4].

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In order to take advantage of noise-like pulses in various applications of laser physics, additional research is required. In this work, we propose a greatly simplified model of noise-like pulses, which is much easier to implement than a full-vector NLSE-based model used earlier[9, 10]. Furthermore, our research offers deeper insight into the nature of noise-like pulses by revealing their stochastic and deterministic properties.

2. GENERATION AND MODELLING OF NOISE-LIKE PULSES

The most common way to generate noise-like pulses is via fibre lasers passively locked due to the effect of non-linear polarisation evolution (NPE). It should also be noted that to date noise-like pulses have also been demonstrated in other types of passively mode-locked lasers, including those using saturable absorbers made of single-walled carbon nanotubes[27], as well as topological insulators[28] and non-linear optical loop mirrors (NOLM)[13,29] or non-linear amplifying loop mirror (NALM)[30-32].

In relatively short lasers (with few-meter-long cavities), both noise-like and 'regular' (single-scale) pulses (as well as some transitional or intermediate regimes between these two[33]) may be generated at different settings of the intra-cavity polarisation controllers or the pump power level. In contrast, elongation of the fibre laser cavity to several hundred metres or several kilometres leads to predominant generation of noise-like pulses[34, 35]. Cavity elongation[36] and increasing pump power[37] are the most straightforward and efficient ways to boost the output pulse energy in passively mode-locked lasers, therefore noise-like pulses are becoming the focal point of research conducted by many groups around the world.

Numerical modelling of noise-like pulse generation in fibre lasers is usually based on the generalised non-linear Schrödinger equation (GNLSE)[35] or on a system of simultaneous equations for polarisation components of the intra-cavity radiation[2]:

$$\frac{\partial A_x}{\partial z} = i\gamma\left\{\left|A_x\right|^2 A_x + \frac{2}{3}\left|A_y\right|^2 A_x + \frac{1}{3}A_y^2 A_x^*\right\} + \frac{g_0/2}{1+E/\left(P_{sat}\cdot\tau\right)}A_x - \frac{i}{2}\beta_2\cdot\frac{\partial^2 A_x}{\partial t^2} \quad (1)$$

$$\frac{\partial A_y}{\partial z} = i\gamma\left\{\left|A_y\right|^2 A_y + \frac{2}{3}\left|A_x\right|^2 A_y + \frac{1}{3}A_x^2 A_y^*\right\} + \frac{g_0/2}{1+E/\left(P_{sat}\cdot\tau\right)}A_y - \frac{i}{2}\beta_2\cdot\frac{\partial^2 A_y}{\partial t^2} \quad (2)$$

where A_x and A_y are the polarisation components of the field envelope, z is the longitudinal coordinate along the fibre, t is the time in the retarded frame of reference, γ and β_2 are non-linear and dispersion coefficients correspondingly, g_0 and P_{sat} stand for unsaturated gain coefficient and saturation power for the active fibre.

Equations (1, 2) describe propagation of radiation along an active fibre. Assuming g_0=0, one can use the same equations to describe laser pulse propagation inside passive cavity fibre. Polarisers, polarisation beam splitters and polarisation controllers (PC) are modelled using 2 x 2 matrices, the PC matrices being unitary[2].

As it was shown earlier[38, 39], simple tuning of intra-cavity PC settings can be used to switch between various lasing regimes, making it possible to use a single experimental configuration to study lasing regimes (both noise-like and 'regular') with widely different pulse characteristics such as energy, pulse duration, and spectral width. Figure 1 illustrates variability of pulse energy in different lasing regimes that were found in simulations of NPE lasers at different settings of intra-cavity PCs at constant pump power level. In particular, Fig. 1 (a) and (b) show probability density functions for output pulse energies in noise-like and 'regular' lasing regimes respectively. These regimes were found in simulations through random changing of settings of intra-cavity polarisation controllers similar to our previous studies[38,39]. Figure 1 (c) and (d) illustrates scatter of both output and intra-cavity pulse energies achievable in NPE mode-locked lasers in noise-like and 'regular' lasing regimes respectively. It can be seen from Fig. 1 that NPE provides an exceptional freedom of laser tuning allowing one to change both parameters of artificial saturable absorber and output coupling ratio in pretty wide ranges.

Full numerical approach based on Eqs. (1, 2) allows one to adequately describe a variety of lasing regimes observed in fibre cavities mode-locked due to NPE[3] as well as NOLM/NALM techniques[31]. However, phase shift introduced by intra-cavity PCs is not amenable to direct measurement in regular experimental implementations. As a consequence, for a valid match between simulations and experiment, one should perform a large series of computations for a range of settings of the intra-cavity PC. For each set of parameters, the entire numerical modelling sequence has to be carried out, including specification of the initial conditions, multiple runs of radiation travel along the cavity, and analysis of the generation regime. In full analogy to the experiment, laser generation only emerges at certain combinations of intra-cavity polarisation element settings and levels of the pumping power launched into the active fibre. Therefore, a considerable amount of calculations carried for randomly chosen polarisation controller parameters does not result in a pulsed generation regime. Thus, for simulations of application of noise-like pulses, a more efficient and simpler model of noise-like pulses is needed. In the present paper, we propose such a model, which is described in the following section.

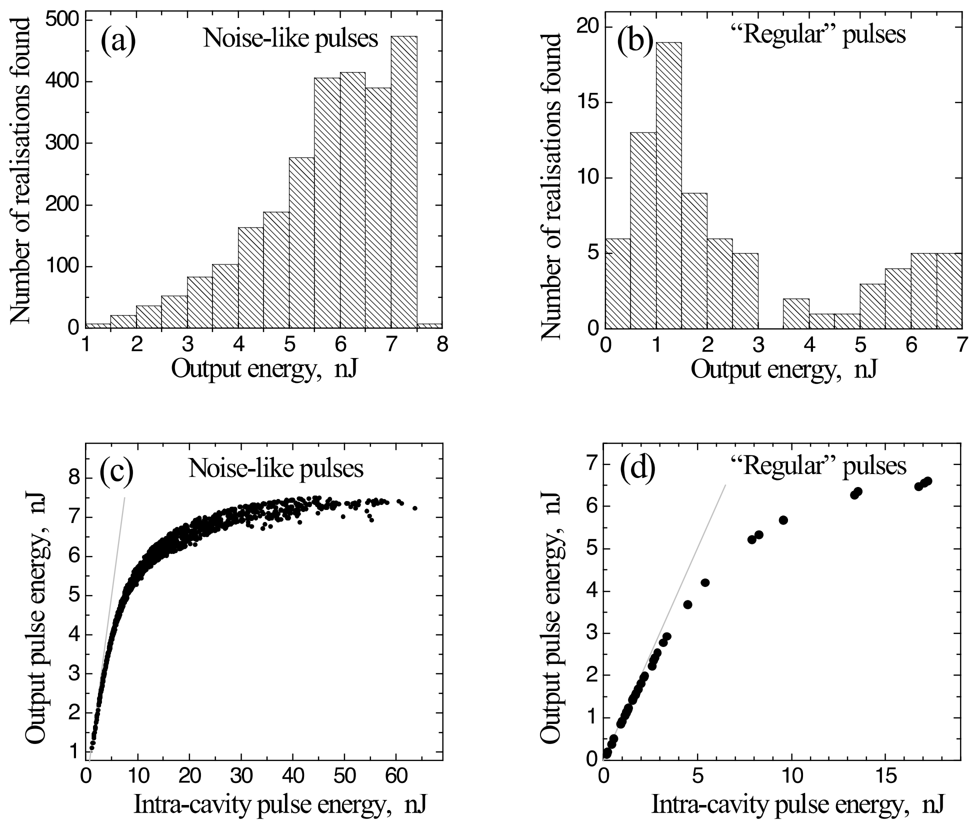

Fig. 1. Variability of pulse energy for (a), (c) noise-like and (b), (d) 'regular' lasing regimes achievable at different settings of intra-cavity PCs of NPE-mode-locked laser at a fixed level of optical pump power (results of numerical simulation).

3. SIMPLIFIED PHENOMENOLOGICAL MODEL

In order to simulate promising applications of noise-like pulses such as frequency conversion, material processing, imaging, *etc* in an efficient way, we propose a model of 'random' pulses constructed in two steps as follows. (i) Let's consider stochastic continuous radiation resulting from superposition of a large number of uncorrelated modes within a given bandwidth. (ii) Now let's form a pulse by multiplying this stochastic quasi-CW radiation by a pulse envelope, *e.g.* Gaussian shape $\exp(-(t/T)^2 \ln 4)$, where T is the pulse full-width at half maximum (FWHM), and ln denotes the natural logarithm. As a result, a random pulse can be represented as

$$A(t) \sim \sqrt{P(t)} \cdot \sum_j A_j \exp(i\omega_j t) \cdot \tag{3}$$

Here, ω_j is the frequency of the j-th mode, t – time, A_j – complex amplitude of the j-th mode, $P(t)$ – temporal profile of "random" pulse, $P(t) = \exp(-(t/T)^2\, 4 \ln 2)$. The phases of complex amplitudes $\arg\{A_j\}$ are taken as independent random variates uniformly distributed from 0 to 2π. Physically 'random pulses' (3) may appear, for example, as a result of temporal shaping of spectrally filtered radiation of a light bulb. Let's compare properties of such "random pulses" with those of noise-like pulses obtained in NLSE-based model of fibre laser.

First of all, let's consider temporal intensity distributions shown in Fig. 2 (a) and (d) for some random realisations of (3) and for some random realisation of noise-like pulses obtained in NLSE-based modelling[38, 39] respectively. Both temporal intensity distributions shown in Fig. 2 (a) and Fig. 2 (d) consist of numerous sub-pulses filling a bell-shaped envelope of a wave-packet. (Note that in this particular case the wave-packet envelope was given a Gaussian form, however any particular pulse shape $P(t)$ known from experiment may be used in the proposed model in Eq. (3).) Sub-pulses inside a wave-packet are located chaotically, having random durations and amplitudes. Note that the duration of these sub-pulses is in the femtosecond range, so that individual sub-pulses cannot be directly resolved by oscilloscope in experiment. In most experimental configurations, the presence of such sub-pulses inside a wave-packet manifests itself as a narrow (femtosecond) peak on ACF. Figure 2 (b, e) shows the ACFs of 'random' pulses (3) and noise-like pulses obtained in NLSE-based modeling[38, 39] respectively. The grey line in Fig. 2 (b) corresponds to a single realisation of ACF whereas the black line appears as a result of ensemble averaging over a large number of random realisations of A_j in Eq. (3). Let's note that both ACFs shown in Fig. 2 (b) and Fig. 2 (e) have a bell-shaped pedestal with a narrow peak on it. Central peaks of both ACFs are shown magnified in Fig. 2 (c) and Fig. 2 (f). It can be readily seen from Fig. 2 (c) and Fig. 2 (f) that there exists some qualitative difference in the height of ACF pedestal between 'random' pulses (3) and noise-like pulses obtained within NLSE-based model (1, 2). In the proposed model of 'random' pulses (3), the height of ACF pedestal is equal to 0.5 (see Fig. 2 (c)), indicating completely random nature of radiation. However, for noise-like pulses obtained within NLSE-based model (1, 2) ACF pedestal is a bit higher (reaching about 0.64 in magnitude), which suggests the presence of some additional mode correlations inherent to noise-like pulses that are not considered by the proposed model (3).

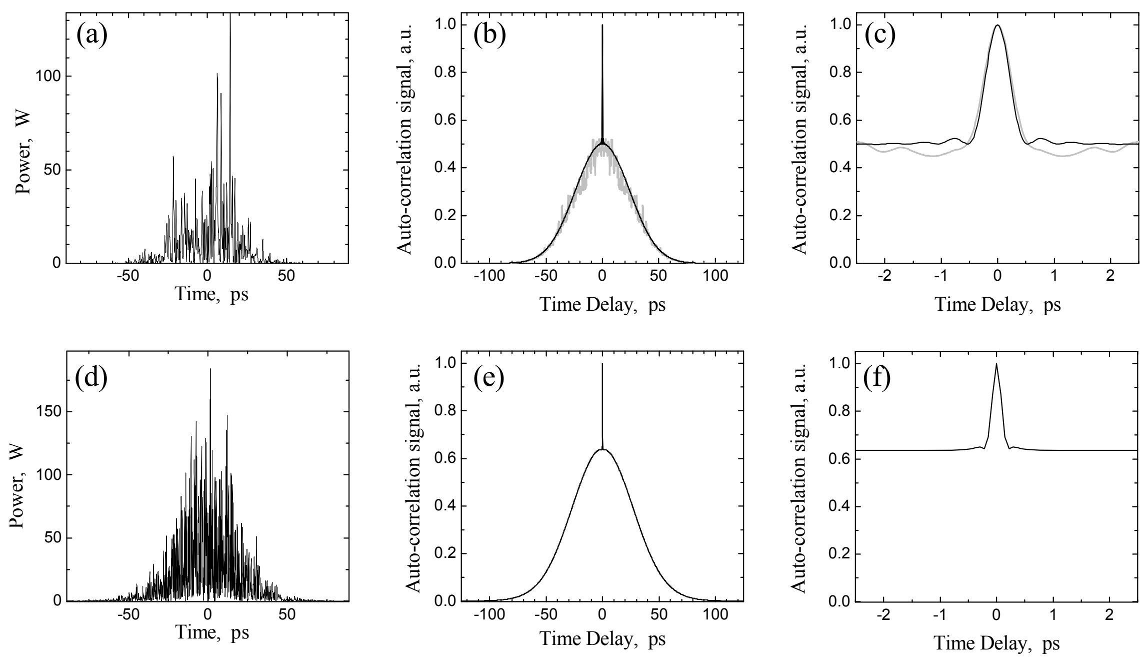

Fig. 2. (a, d) Temporal intensity distributions and (b, c, e, f) intensity auto-correlation functions of 'random' pulses (upper row a, b, c) and noise-like pulses (lower row d, e, f). Panels (c) and (f) zoom into the central ACF peak of graphs (b) and (e) respectively.

Spectra of 'random' pulses (3) and noise-like pulses obtained within NLSE-based model (1, 2) are shown in Fig. 3 (a) and Fig. 3 (c) respectively. In both cases the grey lines show single random realisations of pulse spectrum, whereas the black lines correspond to the results of ensemble averaging. Similarly to the temporal intensity distribution, the optical spectrum of a single pulse is peaky. As a rule, experimentally measured spectra are smooth due to averaging over multiple successive pulses in a pulse train generated by a mode-locked laser. One can easily see totally different forms of ensemble-averaged spectra shown in Fig. 3. Thus, the spectrum of noise-like pulses obtained within NLSE-based model

(1, 2) and shown in Fig. 3 (c) has a bell-shaped form whereas the spectrum of "random" pulses (3) shown in Fig. 3 (a) is rectangular (Π-shaped). It should be stressed that this difference is not a drawback of the proposed model. Indeed, the model (3) can use any spectral shape $|A_j(\omega)|^2$ known from experiment or direct simulation of laser generation. We used a rectangular Π-shaped spectrum in (3) when plotted Fig. 3 (a) only for the sake of simplicity.

Finally, let's consider mode correlations that appear in NLSE-based simulations and in the model of "random" pulses, see Eq. (3). In what follows we'll use the coefficient of mode correlations γ calculated for a pair of modes in simulation, one of them being the central mode of the spectrum and the other having frequency detuning ν from the central spectral mode:

$$\gamma(\nu) = \gamma(A_\nu, A_0) = \frac{\left|\sum A_\nu A_0^*\right|}{\sqrt{\left(\sum |A_\nu|^2\right)\cdot\left(\sum |A_0|^2\right)}} \tag{4}$$

where A_ν and A_0 are complex amplitudes of spectral modes, one (A_ν) with frequency detuning ν from the centre of the spectrum, the other (A_0) is the amplitude of central mode in the generated spectrum. The graphs $\gamma(\nu)$ for "random" pulses (3) and noise-like pulses obtained within NLSE-based model (1, 2) are shown in Fig. 3 (b) and 3 (d) respectively. Since correlation coefficient of any variate with itself is equal to unity, $\gamma(\nu = 0) = 1$. Let's note that both graphs look similar. Adjacent spectral modes of both "random" and noise-like pulses are strongly correlated but correlation coefficient drops rapidly down to 0 in a spectral range much narrower than pulse bandwidth. The width of the spectral interval of correlated modes is equal to the spectral width of pulse envelope both for "random" and noise-like pulses. It should be noted that different modes A_j in Eq. (3) were independent before being multiplied by $P(t)^{1/2}$, which fact was carefully checked in simulations. Thus, mode correlations of "random" pulses (3) appear solely as a result of pulse shaping.

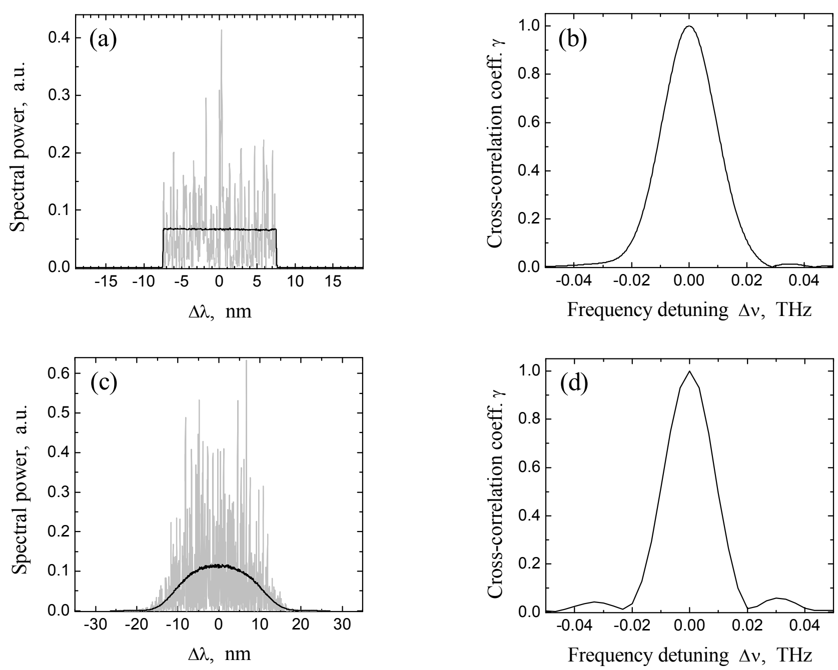

Fig. 3. Spectra (a, c) and coefficient of mode correlations γ(Δν,0) (b, d) of 'random' pulses (upper row a, b) and noise-like pulses (lower row c, d).

4. CONCLUSION

A simple and efficient model of noise-like pulses is proposed. The model allows adequate reproduction of temporal and spectral properties of noise-like pulses as well as correlation between adjacent modes. Within this model one can use any temporal and spectral pulse profiles known from experiment or from exhaustive modelling of noise-like pulse generation in order to simulate applications of noise-like pulses in an easy and efficient way. The question of excessive mode correlation intrinsic especially to intermediate (partially coherent) lasing regimes[6] remains to be addressed in our future work.

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