Modeling of spectral and statistical properties of a random distributed feedback fiber laser

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Abstract: For the first time we report full numerical NLSE-based modeling of generation properties of random distributed feedback fiber laser based on Rayleigh scattering. The model which takes into account the random backscattering via its average strength only describes well power and spectral properties of random DFB fiber lasers. The influence of dispersion and nonlinearity on spectral and statistical properties is investigated. The evidence of non-gaussian intensity statistics is found.

OCIS codes: (140.3510) Lasers, fiber; (140.3490) Lasers, distributed-feedback; (290.5870) Scattering, Rayleigh.

References and links

1. S. K. Turitsyn, S. A. Babin, A. E. El-Taher, P. Harper, D. V. Churkin, S. I. Kablukov, J. D. Ania-Castañón, V. Karalekas, and E. V. Podivilov, "Random distributed feedback fibre laser," Nat. Photonics **4**(4), 231–235 (2010).
2. D. V. Churkin, S. A. Babin, A. E. El-Taher, P. Harper, S. I. Kablukov, V. Karalekas, J. D. Ania-Castañón, E. V. Podivilov, and S. K. Turitsyn, "Raman fiber lasers with a random distributed feedback based on Rayleigh scattering," Phys. Rev. A **82**(3), 033828 (2010).
3. I. D. Vatnik, D. V. Churkin, S. A. Babin, and S. K. Turitsyn, "Cascaded random distributed feedback Raman fiber laser operating at 1.2 μm," Opt. Express **19**(19), 18486–18494 (2011).
4. A. M. R. Pinto, O. Frazão, J. L. Santos, and M. Lopez-Amo, "Multiwavelength fiber laser based on a photonic crystal fiber loop mirror with cooperative Rayleigh scattering," Appl. Phys. B **99**(3), 391–395 (2010).
5. A. E. El-Taher, P. Harper, S. A. Babin, D. V. Churkin, E. V. Podivilov, J. D. Ania-Castanon, and S. K. Turitsyn, "Effect of Rayleigh-scattering distributed feedback on multiwavelength Raman fiber laser generation," Opt. Lett. **36**(2), 130–132 (2011).
6. S. A. Babin, A. E. El-Taher, P. Harper, E. V. Podivilov, and S. K. Turitsyn, "Tunable random fiber laser," Phys. Rev. A **84**(2), 021805 (2011).
7. M. Pang, S. Xie, X. Bao, D. P. Zhou, Y. Lu, and L. Chen, "Rayleigh scattering-assisted narrow linewidth Brillouin lasing in cascaded fiber," Opt. Lett. **37**(15), 3129–3131 (2012).
8. Y. J. Rao, W. L. Zhang, J. M. Zhu, Z. X. Yang, Z. N. Wang, and X. H. Jia, "Hybrid lasing in an ultra-long ring fiber laser," Opt. Express **20**(20), 22563–22568 (2012).
9. D. V. Churkin, A. E. El-Taher, I. D. Vatnik, J. D. Ania-Castañón, P. Harper, E. V. Podivilov, S. A. Babin, and S. K. Turitsyn, "Experimental and theoretical study of longitudinal power distribution in a random DFB fiber laser," Opt. Express **20**(10), 11178–11188 (2012).
10. A. M. R. Pinto, M. Lopez-Amo, J. Kobelke, and K. Schuster, "Temperature fiber laser sensor based on a hybrid cavity and a random mirror," J. Lightwave Technol. **30**(8), 1168–1172 (2012).
11. J. Nuño, M. Alcon-Camas, and J. D. Ania-Castañón, "RIN transfer in random distributed feedback fiber lasers," Opt. Express **20**(24), 27376–27381 (2012).
12. A. M. R. Pinto, O. Frazão, J. L. Santos, M. Lopez-Amo, J. Kobelke, and K. Schuster, "Interrogation of a suspended-core Fabry-Perot temperature sensor through a dual wavelength Raman fiber laser," J. Lightwave Technol. **28**, 3149–3155 (2010).
13. H. F. Martins, M. B. Marques, and O. Frazão, "Temperature-insensitive strain sensor based on four-wave mixing using Raman fiber Bragg grating laser sensor with cooperative Rayleigh scattering," Appl. Phys. B **104**(4), 957–960 (2011).
14. D. Wiersma, "Disordered photonics," Nat. Photonics **7**(3), 188–196 (2013).
15. I. D. Vatnik, D. V. Churkin, and S. A. Babin, "Power optimization of random distributed feedback fiber lasers," Opt. Express **20**(27), 28033–28038 (2012).
16. H. Cao, J. Y. Xu, D. Z. Zhang, S.-H. Chang, S. T. Ho, E. W. Seelig, X. Liu, and R. P. H. Chang, "Spatial confinement of laser light in active random media," Phys. Rev. Lett. **84**(24), 5584–5587 (2000).

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17. S. Mujumdar, M. Ricci, R. Torre, and D. S. Wiersma, “Amplified extended modes in random lasers,” Phys. Rev. Lett. **93**(5), 053903 (2004).
18. H. E. Türeci, L. Ge, S. Rotter, and A. D. Stone, “Strong interactions in multimode random lasers,” Science **320**(5876), 643–646 (2008).
19. J. Fallert, R. J. B. Dietz, J. Sartor, D. Schneider, C. Klingshirn, and H. Kalt, “Co-existence of strongly and weakly localized random laser modes,” Nat. Photonics **3**(5), 279–282 (2009).
20. P. Stano and P. Jacquod, “Suppression of interactions in multimode random lasers in the Anderson localized regime,” Nat. Photonics **7**(1), 66–71 (2012).
21. Z. Wang, X. Jia, Y. Rao, Y. Jiang, and W. Zhang, “Novel long-distance fiber-optic sensing systems based on random fiber lasers,” Proc. SPIE **8351**, 835142 (2012).
22. C. Vanneste, P. Sebbah, and H. Cao, “Lasing with resonant feedback in weakly scattering random systems,” Phys. Rev. Lett. **98**(14), 143902 (2007).
23. G. A. Berger, M. Kempe, and A. Z. Genack, “Dynamics of stimulated emission from random media,” Phys. Rev. E Stat. Phys. Plasmas Fluids Relat. Interdiscip. Topics **56**(5), 6118–6122 (1997).
24. X. Jiang and C. M. Soukoulis, “Time dependent theory for random lasers,” Phys. Rev. Lett. **85**(1), 70–73 (2000).
25. J. Lü, J. Liu, H. Liu, K. Wang, and S. Wang, “Theoretical investigation on temporal properties of random lasers pumped by femtosecond-lasing pulses,” Opt. Commun. **282**(11), 2104–2109 (2009).
26. J. Andreasen and H. Cao, “Numerical study of amplified spontaneous emission and lasing in random media,” Phys. Rev. A **82**(6), 063835 (2010).
27. X. Wu, J. Andreasen, H. Cao, and A. Yamilov, “Effect of local pumping on random laser modes in one dimension,” J. Opt. Soc. Am. B **24**(10), A26–A33 (2007).
28. Y. Xie and Z. A. Liu, “A new physical model on lasing in active random media,” Phys. Lett. A **341**(1-4), 339–344 (2005).
29. G. P. Agrawal, *Nonlinear Fiber Optics* (Academic Press, 2001).
30. C. E. Preda, G. Ravet, A. A. Fotiadi, and P. Mégret, “Iterative method for Brillouin fiber ring resonator,” in CLEO/Europe 2011 Conference, OSA Technical Digest (Optical Society of America, 2011), paper CJ_P27.
31. S. K. Turitsyn, A. E. Bednyakova, M. P. Fedoruk, A. I. Latkin, A. A. Fotiadi, A. S. Kurkov, and E. Sholokhov, “Modeling of CW Yb-doped fiber lasers with highly nonlinear cavity dynamics,” Opt. Express **19**(9), 8394–8405 (2011).
32. A. E. Bednyakova, O. A. Gorbunov, M. O. Politko, S. I. Kablukov, S. V. Smirnov, D. V. Churkin, M. P. Fedoruk, and S. A. Babin, “Generation dynamics of the narrowband Yb-doped fiber laser,” Opt. Express **21**(7), 8177–8182 (2013).
33. D. V. Churkin, S. V. Smirnov, and E. V. Podivilov, “Statistical properties of partially coherent cw fiber lasers,” Opt. Lett. **35**(19), 3288–3290 (2010).
34. S. Randoux, N. Dalloz, and P. Suret, “Intracavity changes in the field statistics of Raman fiber lasers,” Opt. Lett. **36**(6), 790–792 (2011).
35. D. V. Churkin, O. A. Gorbunov, and S. V. Smirnov, “Extreme value statistics in Raman fiber lasers,” Opt. Lett. **36**(18), 3617–3619 (2011).
36. D. V. Churkin and S. V. Smirnov, “Numerical modelling of spectral, temporal and statistical properties of Raman fiber lasers,” Opt. Commun. **285**(8), 2154–2160 (2012).
37. R. G. Smith, “Optical power handling capacity of low loss optical fibers as determined by stimulated Raman and Brillouin scattering,” Appl. Opt. **11**(11), 2489–2494 (1972).
38. A. A. Fotiadi and R. V. Kiyan, “Cooperative stimulated Brillouin and Rayleigh backscattering process in optical fiber,” Opt. Lett. **23**(23), 1805–1807 (1998).
39. J. D. Ania-Castañón, “Quasi-lossless transmission using second-order Raman amplification and fibre Bragg gratings,” Opt. Express **12**(19), 4372–4377 (2004).
40. M. N. Zervas and R. I. Laming, “Rayleigh scattering effect on the gain efficiency and noise of Erbium-doped fiber amplifiers,” IEEE J. Quantum Electron. **31**(3), 468–471 (1995).
41. M. D. Mermelstein, R. Posey, Jr., G. A. Johnson, and S. T. Vohra, “Rayleigh scattering optical frequency correlation in a single-mode optical fiber,” Opt. Lett. **26**(2), 58–60 (2001).
42. S. A. Babin, D. V. Churkin, A. E. Ismagulov, S. I. Kablukov, and E. V. Podivilov, “Four-wave-mixing-induced turbulent spectral broadening in a long Raman fiber laser,” J. Opt. Soc. Am. B **24**(8), 1729–1738 (2007).
43. S. A. Babin, V. Karalekas, P. Harper, E. V. Podivilov, V. K. Mezentsev, J. D. Ania-Castañón, and S. K. Turitsyn, “Experimental demonstration of mode structure in ultralong Raman fiber lasers,” Opt. Lett. **32**(9), 1135–1137 (2007).

1. Introduction

Recently, a concept of a new type of a random laser operating via extremely weak random scattering in a single mode fiber has been proposed and experimentally demonstrated [1,2]. The random distributed feedback (DFB) fiber laser has a mirrorless open cavity with feedback owing to random Rayleigh backscattering (RS) amplified via Raman gain. The laser generates spatially confined spectrally localized radiation in quasi-CW regime. Different random DFB fiber laser systems were realized [3–10] generating in different spectral bands [3–6],

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providing cascaded [2,7], tunable [6], multi-wavelength output [4,5]. The noise level of random DFB fiber lasers could be lower than of conventional lasers [11] making them attractive for telecom applications. Another application is optical sensing [12,13].

In general, theoretical description of random lasers is challenging [14]. Power performances of a random DFB fiber laser could be described within simple power balance model [1,6] providing good prediction for generation threshold, longitudinal generation power distribution [9] and a way to optimization of the laser power performance [15]. Noise properties of a random DFB fiber laser can be also treated within power balance model [11].

To describe spectral and temporal properties of different types of random lasers various models are used. The random generation is represented in terms of sets of modes either of passive or active cavity, localized or extended [14, 16–20]. Other approaches are based on Maxwell's equations combined with the rate equations of a n-level system [21,22]. Numerical methods to solve such systems are varying and include Monte-Carlo simulation of a random walk of photons [23], the finite difference time domain method [24–26], the transfer matrix method [27,28]. None of these models are not applied to the description of the random DFB fiber laser. In general, its spectral, temporal and statistical properties are not investigated neither analytically nor numerically. It is also unknown how spectral and temporal/statistical properties depend on fiber specifications (namely, on dispersion and nonlinearity).

In fiber optics, generalized Schrödinger equation (NLSE) is the most powerful and widely applied method to describe various fiber optics systems [29]. In particular, NLSE-based modeling describes well power, spectral, temporal and statistical properties of quasi-CW fiber lasers with conventional cavities made of point-based mirrors including Brillouin lasers [30], Ytterbitum-doped fiber lasers [31,32] and Raman fiber lasers (RFLs) [33–36].

In the present paper, we adopt NLSEs based model for full numerical description of power, spectral, temporal and statistical properties of the random DFB fiber laser radiation. The simple model which takes into account only an average energy feedback via random Rayleigh backscattering provides an adequate description of the random DFB fiber laser generation properties. We reveal the influence of fiber dispersion and nonlinearity on spectral and statistical properties of random radiation as well as show indications of non-gaussian statistics of generated random radiation.

2. Numerical model

We consider numerically the symmetrical random DFB fiber laser scheme following initial laser design from [1], Fig. 1. The laser cavity comprises 2 spools of fiber of length 41.5 km each. Two pumps (1455 nm) are coupled at the central point of in positive and negative directions. There is no any point based reflectors. The laser generates at 1555 nm.

We use generalized NLSEs with additional terms describing random Rayleigh scattering. Equations can be z-averaged over dispersion walk-off length of the generation and pump waves what results in nulling of phase cross-modulation term (see details in [36]):

$$\frac{\partial A_p^{\pm}}{\partial z} - \frac{1}{v_{gs}}\frac{\partial A_p^{\pm}}{\partial t} + \frac{i}{2}\beta_{2p}\frac{\partial^2 A_p^{\pm}}{\partial t^2} + \frac{\alpha_p}{2}A_p^{\pm} = i\gamma_p\left|A_p^{\pm}\right|^2 A_p^{\pm} - \frac{g_p(\omega)}{2}\left(\left\langle\left|A_s^{\pm}\right|^2\right\rangle + \left\langle\left|A_s^{\mp}\right|^2\right\rangle\right)A_p^{\pm}, \quad (1)$$

$$\frac{\partial A_s^{\pm}}{\partial z} + \frac{i}{2}\beta_{2s}\frac{\partial^2 A_s^{\pm}}{\partial t^2} + \frac{\alpha_s}{2}A_s^{\pm} - \frac{\varepsilon(\omega)}{2}A_s^{\mp} = i\gamma_s\left|A_s^{\pm}\right|^2 A_s^{\pm} + \frac{g_s(\omega)}{2}\left(\left\langle\left|A_p^{\pm}\right|^2\right\rangle + \left\langle\left|A_p^{\mp}\right|^2\right\rangle\right)A_s^{\pm}, \quad (2)$$

where A is complex field envelope, t is a time in a frame of references moving with pump, v_{gs} is a difference between pump and generation Stokes waves inverse group velocities, β_2, α, γ, g are dispersion, linear attenuation, Kerr and Raman coefficients, ε is Rayleigh scattering coefficient, ω stands for frequency. Note that g and ε are operators applied in the frequency space. Sign ± denotes counter-propagating waves, "s" and "p" are used for generation and pump waves, z is a longitudinal coordinate being z = 0 for starting point of propagation and z

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$= L$ at other fiber end, L is the fiber length. Note that for every point of the fiber, longitudinal coordinate value z for "+" wave corresponds to the value of L-z for "-" wave and vice versa. Spontaneous Raman emission was taken into account by using white noise as initial condition [37]. We use following parameters: $\alpha_s = 0.046$ km^{-1}, $\alpha_p = 0.055$ km^{-1}, $\gamma_s = 1.09$ (km*W)$^{-1}$, $\gamma_p = 1.31$ (km*W)$^{-1}$, $g_s = 0.36$ (km*W)$^{-1}$, $g_p = 0.39$ (km*W)$^{-1}$. The dispersion is chosen to be normal: $\beta_{2s} = 20$ ps^2/km, $\beta_{2p} = 35$ ps^2/km, $1/v_{gs} = -2.3$ ns/km. In contrast to conventional lasers, where the generation wavelength is defined primarily by cavity mirrors, modeling of random DFB fiber laser requires taking into account spectral profile of Raman gain. We use parabolic approximation as an example, $g_i(\omega)=g_i - k\omega^2$, where k = 0.0062 ps^2/W/km, $i = s,p$.

Rayleigh backscattering is a random process on a sub-micron scale [38]. To take into account its random nature in NLSE dynamical approach is practically impossible in a laser having tens of kms length, as it is requires integration with micron scale numerical step over all laser length. However, taking into account only an average energy income results in good quantitative description of a random DFB fiber laser power performance within the power balance model [9,15]. The same approach is used to deal with RS in amplifiers [39]. We follow this approach and add an average term proportional to $\varepsilon = 4.5 \times 10^{-5}$ km^{-1} to Eq. (2) (this value may vary depending on fiber NA and fabrication method [40]). Note that we consider only energy income into generation wave whereas corresponding energy depletion of pump wave is considered through linear losses α. Rayleigh scattering induced energy income to pump wave is neglected as it's not amplified. Rayleigh term was calculated in Fourier domain what makes it easy to make ε frequency-dependent by introducing factor $(\omega + \omega_0)^2/\omega_0^2$ for amplitudes which correspond to λ^{-4} law for energy.

We integrated Eqs. (1),(2) along z using iterative approach, i.e. when integrating equations for $A^+_{s,p}$ we used $A^-_{s,p}$ obtained on previous iteration, and vice versa. To integrate Rayleigh term, one has to save optical spectra of generation wave at each integration step to use them at next iteration, that is really memory consuming and cannot be implemented for long lasers operating at high powers. So we save optical spectra only at $N_1 = 50$ z-points during each iteration. Finally, here we do not take into account correlation properties of the random backscattering [41], just adding a random phase factor $exp(i\varphi_0 + i\omega\tau_0)$ with random phase φ_0 and time τ_0 shifts. So we use following mathematical procedure to integrate Rayleigh term:

$$\delta A_s^{+\text{Rayleigh}} = \left(\varepsilon \cdot \int\limits_{z}^{z+\Delta z} \mathrm{d}z' \int\limits_{-\infty}^{+\infty} \mathrm{d}\omega \left| A_s^- \left(L - z' \right) \right|^2 \right) \cdot \frac{A_s^- \left(L - z_{\text{prox}} \right)}{\int \left| A_s^- \left(L - z_{\text{prox}} \right) \right|^2 \mathrm{d}\omega} \cdot \frac{\left(\omega + \omega_0 \right)^2}{\omega_0^2} \cdot e^{i\phi_0 + i\omega\tau_0}, \quad (3)$$

where Δz is an integration step, $\omega + \omega_0$ and ω_0 are optical and carrier frequencies, $z_{prox}(z)$ is a staircase function which approximates z with a set of $N_1 = 50$ steps, each of them is a z-coordinate of the closest point where spectra of "-" wave is saved at previous iteration.

3. Results and discussion

Firstly, we calculate the output laser power as a function of single-side pump power, Fig. 2(a). The simulation predicts a lasing threshold close to 0.8 W in agreement with experimental observations [1,2] and analytical calculations [1,15]. Well above the threshold, the output power grows linearly with pump power that also agrees well with previous experimental observations [1,2]. The low generation slope efficiency (~15% at high pump power) is caused by linear attenuation of the generation wave as the fiber length is larger than typical amplification length in the system [1,9]. This fact becomes clear from Fig. 2(b). Indeed, the generation power reaches its maximum at z ~12 km. At z > 12 km the generation wave is attenuated as linear losses are higher than Raman gain due to pump power depletion. The numerically calculated within NLSE-based model power distributions agree qualitatively well with experiment and analytical calculations made within power balance equations [9].

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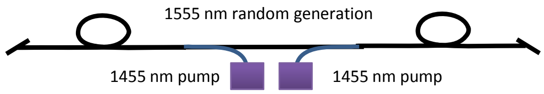

Fig. 1. Random DFB fiber laser scheme.

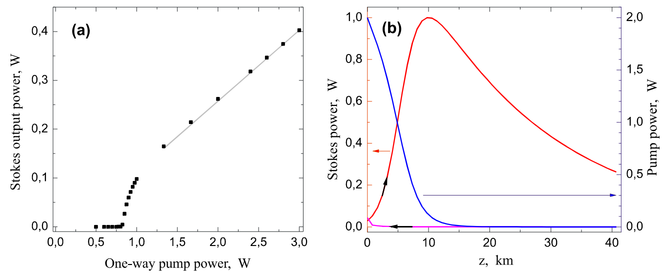

Fig. 2. (a) Output laser power vs. one-side pump power. (b) Longitudinal power distribution for pump (blue line) and generation (red line for "+" and magenta line for "–" generation waves).

Typical averaged spectra of generated Stokes wave are shown in Fig. 3(a). The lasing spectrum is much narrower than initial Raman gain spectral profile in qualitative agreement with experimental data [1]. Thus a simple model taking only an average random Rayleigh backscattering strength provides a good description of both power and spectral properties of random DFB fiber laser. The higher the generation power, the more pronounced is the nonlinear spectral broadening, Fig. 3(b). We anticipate that such spectral broadening is owing to Kerr nonlinearity and dispersion interplay similar to the processes in the conventional mirror-based fiber lasers [32, 33, 42]. Indeed, in our system the generation spectrum becomes narrower for systems with higher dispersion, Fig. 3(c), that could be understand as dispersion prevents to different spectral components to interact nonlinearly in effective way. Similarly, changing the nonlinearity at fixed pump power and fixed dispersion, we observe that spectrum becomes broader in the systems with larger nonlinearity, Fig. 3(c). There are no up to date any experimental data to compare our findings of dispersion and nonlinearity influence on spectrum width. The proper dispersion and/or nonlinear management could be a practical tool to change the spectral properties in real random DFB fiber laser systems.

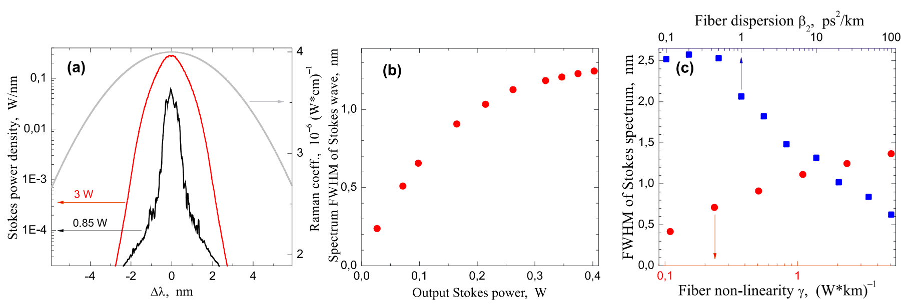

Fig. 3. (a) Raman gain profile (grey) and spectrum for generated "+"-wave at laser output as a function of $\Delta\lambda = \lambda$–1555 nm for two pump powers of 0.85 W (black) and 3 W (red); (b) lasing spectrum bandwidth (rms) depending on generation power (c) spectrum width at P = 2 W at different dispersion and nonlinearity value.

Finally, we calculate temporal and statistical properties. The intensity dynamics reveals highly stochastic nature of the radiation, Fig. 4(a). The typical time scale of fluctuations is ~5 ps, Fig. 4(b), so intensity fluctuations could be hardly measured in real-time using conventional oscilloscopes. Measurements in bandwidth of 40 GHz which could be achieved in state-of-the-art oscilloscopes will result in averaging of actual intensity dynamics, Fig. 4(a) (black). Surprisingly the radiation statistics is not completely Gaussian in random DFB fiber laser. The lower the dispersion, the more non-exponential is intensity probability density function (pdf), Fig. 4(c), revealing probable correlations in radiation. The background level of intensity autocorrelation function (ACF) is higher than 0.5, Fig. 4(b), confirming that the radiation is not completely stochastic. The intriguing question of non-gaussian intensity statistics in the radiation of the random DFB fiber laser has to be further investigated. Note that in conventional mirror based laser cavities non-gaussian intensity statistics is previously reported [33–35] arising from partial correlations between different longitudinal modes, which are well-defined (but still strongly fluctuating and broad) in those systems [43]. Intensity dynamics and statistical properties of random DFB fiber laser are not studied experimentally up to date.

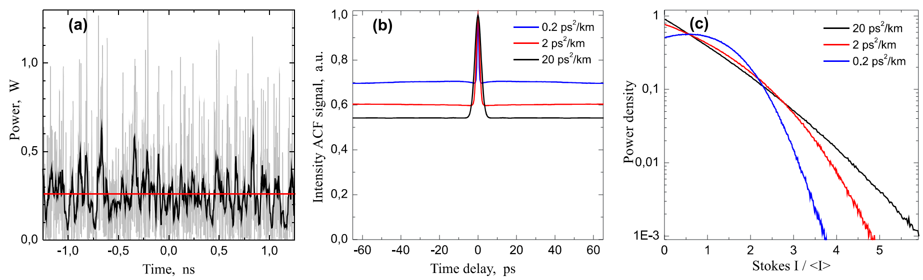

Fig. 4. (a) Typical intensity dynamics (grey shows original simulated data, black – smoothed with a bandwidth of 40 GHz, red – average lasing power level), (b) Intensity ACF and (c) intensity pdfs for different fiber dispersions. Pump power is 2 W on all graphs.

4. Conclusion

For the first time a full numerical modeling of random DFB fiber laser based on Rayleigh scattering are performed with the use of generalized NLSE. It is shown that to describe in general the random DFB fiber laser generation properties, random distributed feedback can be taken into account via average energy income only without taking into consideration the random strength of the scattering on micron scales and its coherence properties. Calculated generation power and its longitudinal distribution as well as optical spectrum are in good qualitative agreement with previous experimental results. It is shown that increasing the dispersion or decreasing the nonlinear coefficient leads to the narrower generation spectrum providing a possibility to spectral management of random DFB fiber laser generation. Temporal and statistical properties of radiation are also studied. The intensity statistics and intensity auto-correlation function reveal non-gaussian statistics of the random DFB fiber laser radiation.

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